GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: November 10, 1976

Project Title: Bi-Material Fracture Program

Project No: E-23-623

Project Director: Dr. S. Atluri

Sponsor: HQ Air Force Flight Test Center; Edwards AFB, CA 93523

Agreement Period: From 10/1/76 Until 12/30/77

Type Agreement: Contract No. F04611-76-C-0078

Amount: $50,433

Reports Required: Program Plan; R&D Status Report; Financial Forecast vs Actual Report; Computer Programming Manual Cracked Elements User's Manual; Computer Program; Final Technical Report

Sponsor Contact Person (s):

Technical Matters

Dr. Randall L. Peeters
AFFTC(AFSC)
ATTN: AFRPL/MKPB
Edwards AFB, CA 93523
(714)553-2796

Contractual Matters
(thru OCA)

Mr. Sam Renneker
ONR Resident Representative
Georgia Institute of Technology
Room 325, Hinman Research Building
Atlanta, GA 30332
(404)526-4374/4213

Defense Priority Rating: DO A-6 under DMS Reg. 1

Assigned to: Engineering Science and Mechanics (School/Laboratory)

COPIES TO:

Project Director
Division Chief (EES)
School/Laboratory Director
Dean/Director—EES
Accounting Office
Procurement Office
Security Coordinator (OCA)
Reports Coordinator (OCA)

Library, Technical Reports Section
Office of Computing Services
Director, Physical Plant
EES Information Office
Project File (OCA)
Project Code (GTRI)
Other

CA-3 (3/76)

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: 1/23/79

Project Title: Bi-Material Fracture Program

Project No: E-23-623

Project Director: Dr. S. Atluri

Sponsor: HQ Air Force Flight Test Center: Edwards AFB, CA 93523

Effective Termination Date: 6/30/78 (R&D Perf. Period); 9/29/78 (Contract Period)

Clearance of Accounting Charges: ---

Grant/Contract Closeout Actions Remaining:

Govt.

4

X Final Invoice and Closing Documents

_ Final Fiscal Report

_ Final Report of Inventions

X Govt. Property Inventory & Related Certificate

X Classified Material Certificate

_ Other ____________

ONR is ACO

Assigned to: Engineering Science & Mechanics (School/Laboratory)

COPIES TO:

Project Director
Division Chief (EES)
School/Laboratory Director
Dean/Director—EES
Accounting Office
Procurement Office
√Security Coordinator (OCA)
Reports Coordinator (OCA)

Library, Technical Reports Section
Office of Computing Services
Director, Physical Plant
EES Information Office
Project File (OCA)
Project Code (GTRI)
Other ____________

CA—4 (3/76)

E23-623

R & D Status Report

on

Bi-Material Fracture Program

under contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan,
Post-Doctoral Fellow

S. N. Atluri,
Project Director

May 1977

As planned in the program schedule the incorporation of homogeneous cracked element program, for three-dimensional fracture problems, into TEXGAP-3D code has been successfully completed. It has been found to be compatible with the TEXGAP-3D code. The stiffness matrices of the four types of homogeneous cracked singular elements can be generated by element cards using identifications such as CRACK 1, CRACK 2, CRACK 3 and CRACK 4.

The computer coding for thermal effects has also been completed. The TEXGAP-3D code provided to us, at present, has the capability of handling only constant temperature level problems, though some provisions are made in it for future incorporation of linear variation of temperatures between corner nodes of the element (for example the array TEMPER (8)). Thus, in the present formulation of thermal problems in cracked elements, provisions for a thermal gradient within the element is also incorporated. This task is essentially the computation of an equivalent nodal force vector corresponding to the specified temperature distribution in the crack element. The computer coding was done carefully and is compatible with the TEXGAP-3D program such that when the TEXGAP-3D program is modified to handle problems with linear variation of temperature between nodes, no modification is necessary in the cracked element program.

The effort to incorporate crack face pressure distribution for homogeneous cracked element is underway and will be completed in the near future. The equivalent nodal force vector will be coded so as to be compatible with TEXGAP-3D program.

After extensive literature survey, the attention was focused on deriving explicit (direct) forms of stresses and displacements, plane strain bi-material fracture problems, in terms of stress-intensity factors and the material properties of both the materials. Such a derivation has already been completed and is being checked again for its arithmetic. This direct form of the near-field displacements and stresses would be of great value in the proper choice of field variables namely, arbitrary interior displacements, inter-element boundary displacements and the element boundary tractions for a bi-material cracked hybrid element.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri,
Project Director

June 1977

The task of computer coding the thermal effects in the homogeneous cracked element program has been completed. This was accomplished by considering the thermal effects as an initial strain problem and computing the equivalent nodal force vector corresponding to the specified temperature distribution in the crack element. Though the TEXGAP-3D code provided to us, at present, is capable of handling only constant temperature level problems, the flexibility of the computer code developed for thermal problems in homogeneous cracked element has been kept such that it could handle problems with linear variation of temperature between corner nodes without any further modification. The computed equivalent nodal force vector was then checked for its symmetries and force equilibrium conditions and it satisfies both of them very well. Then it was added to the TEXGAP-3D code and a small trial problem with thermal effects was run to check its compatibility with the TEXGAP-3D code.

The task of incorporating crack face pressure distribution for homogeneous crack element has also been completed. Based on the previous experience, it is enough if we compute the nodal forces corresponding to the crack face pressure distribution for homogeneous crack element in the same way as we would compute for a regular element. Thus the TEXGAP-3D code has been modified such that it would compute the nodal forces in the same way as it would for a regular element, if there is a pressure distribution on the crack face of the crack element. In other words, to compute the nodal forces corresponding to the pressure distribution on the crack face of the crack element, the program would compute through the subroutines PRLOAD and BIGPR. These changes have already been made in the TEXGAP-3D code and a trial problem with crack face pressure distribution is being run presently.

After completing the derivation of explicit (direct) forms of stresses and displacements in terms of stress-intensity factors and the material properties for a plane strain bi-material fracture problems, attention is now being focused on the assumptions of arbitrary interior displacements, inter-element boundary displacements and the element boundary tractions for bi-material crack hybrid element. This task would be completed in the near future and the computer coding for bi-material crack hybrid element would be started.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program Under Contract

AFRPL F04611-76-C-0078

Submitted by:

K. Kathiresan
Post Doctoral Fellow

Satya N. Atluri
Project Director

July 1977

As mentioned in the previous status report, the task of computer coding the thermal effects and crack face pressure distribution for three-dimensional homogeneous crack element has been completed. The temperature effects are incorporated in the homogeneous crack element itself. When a specific value of $\alpha\Delta T$ is given in the data, homogeneous crack element automatically computes the equivalent nodal forces corresponding to the initial strain due to temperature loading and stores them in the load vector. In the case of crack face pressure distribution, controls have been so arranged that the subroutines PRLOAD and BIGPR would compute the corresponding nodal forces and store them in the load vector as usual. Efforts were made to check the incorporation and compatibility of the computer coding of the above two tasks. Two simple sample problems, one for each case, were devised to check the incorporation and compatibility. The check problems for thermal effects and crack face pressure distribution have been completed and found that the incorporation and compatiblity of the computer coding is very successful.

The derivation of direct forms of stresses and displacements in terms of the stress-intensity factors and material properties for a plane strain (or plane stress) bi-material fracture problems was completed and it was rederived and checked for its accuracy. Using these stresses and displacements and the experience of the homogeneous crack element program, the preliminary assumptions of arbitrary interior displacements inter-element boundary displacements and the element boundary tractions for bi-material hybrid crack element have been already made. Using these three assumed field variables, the computer coding for three-dimensional hybrid displacement crack element for bi-material fracture problems has been initiated.

E-23-623

R & D STATUS REPORT

ON

BI-MATERIAL FRACTURE PROGRAM UNDER CONTRACT

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

August 1977

The development and computer coding of thermal effects and crack face pressure distribution for three-dimensional homogeneous crack element have been completed. The compatibility of the development and coding with TEXGAP-3D code has been thoroughly checked and the incorporation of homogeneous crack element with thermal and pressure effect capability into TEXGAP-3D was very successful. Small trial problems with thermal and crack face pressure effects have been run and the solutions are found to yield accurate results. The program will be run for bigger size problems with thermal and crack face pressure distribution in near future.

As mentioned in the previous R & D Status Report, the effort to computer code the three-dimensional hybrid displacement crack element for bi-material fracture problems has been initiated and is underway. The three field variables in the hybrid displacement model for bi-material fracture problems are assumed in an identical fashion as that of homogeneous element model. These three field variables include proper singualr solutions of displacements and stresses corresponding to the bi-material fracture problems. The inter-element boundary displacement has been choosen such that it will be compatible with the neighbouring regular elements and also that it would have proper singular variations of displacements on the other boundaries. This bi-material program also will be developed identical to that of homogeneous material program so that the incorporation and utilization would also be identical.

E-23-623

R. &. D. Status Report

on

Bi-Material Fracture Program

Under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

September 1977

As scheduled in the program plan of this contract, optimization of the coding and incorporation of three-dimensional hybrid-displacement homogeneous crack element with thermal and pressure effect capability into the TEXGAP-3D coding have been successfully completed. Though the task of optimization of the coding of homogeneous crack element, for its computer memory requirements and execution time to generate stiffness matrices of singular elements, is essentially completed, efforts are still being made to further optimize wherever it is possible. To check the development, incorporation of the thermal and crack pressure effects for homogeneous crack element and its compatibility with the TEXGAP-3D code, small trial problems were run and the solutions are found to yield accurate results. To check the reliability and accuracy of the incorporation of homogeneous crack element program into TEXGAP-3D, fairly large size complex fracture problems need to be solved. The datas for such problems with thermal and crack pressure effects are now being generated and the execution of the program for these problem will be carried out in near future.

The computer coding for obtaining the condensed stiffness matrices for three-dimensional bi-material crack elements rising hybrid-displacement finite element model is underway. Similar to the homogeneous crack element, the basic element for bi-material crack element will be a 20 node isoparametric brick element with proper bi-material singularities. As the stresses for bi-material problems are singular $(1/\sqrt{r})$ and possess a logarithmic oscillation, transformation to eliminate $1/\sqrt{r}$ singularity and efficient numerical integration schemes will be used. In the case of bi-material fracture problems, there is an additional parameter involved namely the bi-material constant ϵ. Care must be exercised in this case to define and compute this parameter properly before any of the stiffness matrices of bi-material crack elements are computed.

E 23-623

R & D Status Report

on

Bi-Material Fracture Program

Under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

October 1977

As indicated in the previous status report, bi-material problems involve an additional parameter called bi-material constant ϵ which is a function of Poisson ratio and shear modulus of both the materials. This parameter has to be defined and computed properly before attempting to compute the stiffness matrices of bi-material singular elements. After studying the TEXGAP-3D program carefully, it was found that the best place for defining and computing the bi-material constant ϵ would be in the subroutine ELDATA in OVERLAY SETUP. It has been arranged presently that the two materials involved in bi-material fracture problems would be defined during the element definition control cards. The control card would look as follows: BIMAT, M1, M2 where M1 and M2 are the material numbers of material 1 and material 2 respectively in bi-material fracture analysis. Though we are forced to define these bi-materials during element definition, the flexibility of defining the BIMAT control card anywhere in the element definition is provided. In other words, the BIMAT control card may occur anywhere in element definition or boundary conditions but before END, ELEMENTS card. A separate subroutine called BICON is now added to compute the bi-material constant ϵ using the material properties of the two materials.

The computer coding of the three-dimensional bi-material crack element using hybrid-displacement finite element model to obtain the condensed stiffness matrices is underway and will be completed in near future. Once the condensed stiffness matrices are obtained, several checks for their properties, like satisfaction of equilibrium conditions at the element level and the inherent directional symmetries and antisymmetries, etc. will be made.

The homogeneous crack element program has now been stored in the disk at LBL, Berkeley, California and efforts to compile and execute complex fracture problems are being made.

E 23-623

R & D Status Report

on

Bi-Material Fracture Program

Under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

November 1977

The computer coding of the three-dimensional bi-material crack element using hybrid-displacement finite element model to obtain the condensed stiffness matrices is almost complete. The modifications needed for bi-material fracture analysis have been incorporated and the stiffness matrices have been generated. Now efforts are being concentrated on the accuracy of the matrices. Checks like satisfaction of equilibrium conditions at the element level etc., are being made. After the test of accuracy and other check, practical three-dimensional bi-material fracture problems will be attempted using the developed computer code.

With respect to the compilation and execution of homogeneous three-dimensional crack element program in CDC-7600 machine at LBL, Berkeley, California, we are facing lot of difficulties. The difficulties we are facing may be enumerated as follows. In the beginning, it took lot of effort to even store the TEXGAP-3D program at LBL disk, since there was no facility available to transmit the code through magnetic tapes and it has to be transmitted only through punched card deck. After trying four times, reading about 11000 cards each time, we finally succeeded in storing the program at LBL disk. We expected that the Job Control Language (JCL) of CDC-7600 at LBL would be similar to the Job Control Language (JCL) of CDC-6600 at Georgia Tech, probably with slight differences. Unlike our expectation, though the structure of the JCLs are same, they were quite different in terms of terminology and the parameters involved in each control card. We neither have CDC-7600 control language manuals nor anybody who knows the JCL of CDC-7600 at Army Corps of Engineers in Atlanta. So, we are presently trying to compile and execute the TEXGAP-3D program with extensive consulting from consultants at LBL, Berkeley and Army Corps of Engineers at various places in the country. Until recently we have been working with the help of Mr. Larry Mitchell of Army Corps of

Engineers at Wilmington, N. C. He helped us in successfully compiling the TEXGAP-3D program, but we are unable to execute it. This is due to the reason that the TEXGAP-3D program is an overlaid program and Mr. Mitchell didn't have any experience in overlaid programs. Then we came to know that Mr. Bob Williams of Army Corps of Engineers at Kansas City has experience in overlaid program and we are presently working with his help and close contact.

Moreover, the version of TEXGAP-3D program which we have (given to us about eight months back) seems to have some bugs in it. The execution was terminated in subroutine PSTRES due to some undefined variable or trying to reach a subscript out of range of a dimensioned variable. For this we contacted Dr. Bob Dunham of Pacifica Technology and he said he also faced similar troubles in the case of only one principal stress problems (the other two principal stresses being zero) (as in the case of uniaxial tension) and advised us to skip some computations in subroutine PSTRES.

Another handicap is that CDC-7600s low speed terminal channel (TTY) is closed for us during most of the daytime since it is reserved only for ERDA related jobs. Similarly the high speed channel for COPE machine also is busy frequently. The above mentioned difficulties have slowed down the progress of the work considerably. But Mr. Edward Gutwald and Mr. Mardy Counts of Atlanta, Mr. Bob Williams of Kansas City and Mr. Larry Mitchell of Wilmington, N. C., are giving their best possible help to accelerate the work. With the help of above Army Corps of Engineers, the program will be successfully executed in very near future and the task of solving complex fracture problems will be taken up immediately.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

Under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

December 1977

As mentioned in the previous status report, two major difficulties were faced with respect to the execution of TEXGAP-3D program with three-dimensional homogeneous crack element incorporated in it. The first one being some undefined variables in the subroutine PSTRES which terminated the execution. This trouble was overcome after discussing with Dr. Robert Dunham of Pacifica Technology who advised us to skip some computation and EQUIVALENCE some of the variables in the subroutine PSTRES. The second one was the unfamiliarity with the JCL of CDC-7600 at LBL. This also was overcome by extensive help from Army Corps of Engineers at Atlanta, Wilmington and Kansas City. Then the program was successfully executed but something was going wrong in the incorporated crack element which yielded wrong answers. After extensive investigation, we found that the CDC-7600 was computing A**J (where A = 0.0 and J = 0) as 0.0 whereas it should have been 1.0 (the Georgia Tech CDC-6600 computes it correctly as 1.0). This may a system error of CDC-7600 at LBL. After fixing this bug, the program was executed successfully with identical results. Efforts to solve fairly complex problems are already underway and the solutions may be expected soon.

The computer coding of the three-dimensional bi-material crack element using the hybrid-displacement finite element model to obtain the condensed stiffness matrices is completed. Some preliminary checks such as satisfaction of equilibrium conditions at the element level and accuracy of recomputation of stress-intensity factors k_1, k_2 and k_3 assuming them as 1.0 initially were made and they yielded very satisfactory results. Simultaneously, effort to incorporate the thermal and crack face pressure effects in the bi-material crack element is also being made. Once this is completed example problems using the developed bi-material crack elements will be solved in near future.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

Under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

January 1978

After debugging the computation of A**J type of statement (CDC-7600 at LBL computes it as 0.0 whereas it should have been 1.0 for values of A = 0.0 and J = 0, see status report of December 1977), the program executed a small sample problem successfully. A fairly large semi-elliptical surface flaw problem was attempted next. While attempting to solve this problem, it was found out that the (old) TEXGAP-3D version cannot solve the present problem because of the following reason. Since the boundary of the crack is an ellipse, the elements containing the crack surface were modelled by BRICK and PRISM elements. In those elements there are many PRISM elements having the same first node numbers. Thus it is impossible, using old TEXGAP-3D version, to apply the (symmetry) boundary condition on these (PRISM) elements because the first node number of the elements must be unique. In order to solve this difficulty, two alternatives are now being attempted. First is to assume a small through hole in the structure and convert the PRISM elements into BRICK elements and then solve the problem. The solution in this case will be close to the solution of the problem without the through hole as long as the diameter of the hole is assumed to be very small. Second is to incorporate the three dimensional crack element, with all the modification as in old TEXGAP-3D, into the new TEXGAP-3D which was provided to us recently and is capable of handling the boundary conditions in terms of nodal points as well as based on elements. These two tasks are underway and the results will be reported in next month's report. The three-dimensional bi-material crack elements with thermal and crack face pressure effects has been incorporated in Georgia Tech computer and small trial problems have been run successfully using this code. Large bi-material crack problems will be run in near future in LBL CDC-7600 computer once the homogeneous crack program is successfully completed.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

Under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

February 1978

In the previous status report, the difficulties that were faced in solving a fairly large complex problem of semi-elliptical surface flaw in a tension plate were given. Two methods were proposed, first to introduce a small through hole to convert all the PRISM elements to BRICK elements and second to carry out all the necessary modifications to incorporate the three-dimensional homogeneous crack element into the latest version of TEXGAP-3D provided to us recently, to solve these difficulties. Both of these tasks were carried out and the results are very successful. The problem with the introduction of a small through hole gave excellent results of stress-intensity factors. The latest TEXGAP-3D has been modified already incorporating the crack element and a small trial problem was run which yielded identical result as the old TEXGAP-3D version with the crack element.

The latest version of TEXGAP-3D with the crack element will be loaded in CDC-7600 at LBL as soon as possible and the above mentioned semi-elliptical surface flaw in a plate problem will be solved using this code which is capable of treating the boundary conditions in terms of nodal points as well as on the element basis and the solution will be compared with the solution of the same problem (with a through hole in the plate) by old TEXGAP-3D. Simultaneously, the latest TEXGAP-3D with three-dimensional bi-material crack element will also be loaded in CDC-7600 at LBL. Once these are successfully loaded, several large problems (homogeneous as well as bi-material) will be solved to make sure that the coding is complete and accurate. The writing of the final technical report will be initiated as soon as the above mentioned tasks are completed.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

Under Contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

March 1978

The difficulties that were mentioned in the previous R & D Status Report with regard to the execution of TEXGAP-3D stored at LBL CDC-7600 have been overcome. The TEXGAP-3D program stored at LBL CDC-7600 has now been used for several complex problems and they yielded excellent results. We intend to solve two or three more complex problems with different geometry, for example a thick-walled pressure vessel with a semi-elliptical surface flaw problem. The solutions of the problems, which we intend to solve, will assure the accuracy and reliability of the modification made in TEXGAP-3D code and the application of incorporated crack element for different complex geometry of the structure. The turn-around time is very long at LBL CDC-7600 for big jobs like the present and it takes days before the output comes back and this affects the progress of the work considerably. However the above mentioned problems will be completed in very near future.

We made a rough calculation for solving a complex geometry bi-material problem in the LBL CDC-7600 using the TEXGAP-3D code with the bi-material crack element. But, such problems cannot be solved in LBL CDC-7600 machine. The reasons are that the problems we wanted to solve are as complex as the homogeneous problems we solved already and for bi-material problems the portion of the structure which has to be modeled is twice that of the homogeneous problem. This problem simply cannot be solved because of the limitation of dimension of ELPA in ZIPP3 OVERLAY to 131000 in the secondary memory. Moreover we faced some difficulty while solving homogeneous problems that the TEXGAP-3D code gives erroneous results or terminates the execution due to a pivotal value of 0. for diagonal term in ZIPP3 OVERLAY when small size elements are used near the crack front. Thus we decided to solve fairly simple bi-material problems. But these can be carried out Georgia

Tech Computer itself. Thus the bi-material problems will be executed at Georgia Tech and the results will be given in the technical report. The writing of the technical report has been initiated and is underway. This would include all the theoritical formulations, user mannual for homogeneous and bi-material crack problems and sample input and output of problems.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

under contract

AFRPL F04611-76-C-0078

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

April 1978

With the code incorporation of special singular elements for homogeneous as well as bi-material problems and modification of TEXGAP-3D code completed, several trial problems are being solved for homogeneous and bi-material cases whose input and output will be added in the technical report along with the user mannual. Several simple problems have been solved already and the results were found to be excellent. In order to show the versatility of the application of the incorporated special singular element, and also as a thorough check, some very complex problems are being modeled and solved. One of them is a single corner crack problem, where the crack is at the nozzle-cylinder junction. This is a very complex problem, especially so due to the presence of the corner crack and this represents an important catagory of problem to test the computer coding. The finite element breakdown and the grid generation are being done and the problem would be solved in LBL CDC-7600 in near future. Another problem is that of cylindrical shell with an inner semi-elliptical surface crack. For this problem, thermal shock effects are also being included, the breakdown and grid generation are being done and will be solved in near future.

As mentioned in the previous R & D status report, simple bi-material problems are solved at Georgia Tech. The writing of the user mannual for using homogeneous and bi-material crack elements and the final technical report are underway. About half of this task has been completed and they will be completed in near future. All the test problems described above will be incorporated in the technical report.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

under contract

AFRPL F04611-76-C-0078

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

May 1978

The finite element modeling of the complex problem of a corner crack at nozzle-cylinder junction has been done. This was done by developing a small separate program to breakdown the structure. Before running the problem in LBL CDC-7600, we wanted to make several checks regarding the accuracy of this program which generates the finite element breakdown. This is essential from the point of view of running the problem successfully and obtaining the correct solution of stress-intensity factor values. Most of the development and checking of this small program has been done. After making some final checks, the problem will be executed in LBL CDC 7600 in a week. The result of the running of this problem will be reported in the next month's R & D Status Report. The problem of a cylindrical shell with an inner semi-elliptical crack and thermal effects has been modeled already and the data is ready to be run. Since the nozzle-cylinder problem required lot of work and attention, this problem was not run, but will be run before or with nozzle-cylinder problem.

The writing of the technical report and the user manual are underway. The technical report will contain the theoretical formulation of the hybrid displacement finite element model, the development of field functions for homogeneous and bi-material crack elements, the development of near field solutions of stresses and displacements for homogeneous and bi-material problems and the relevant numerical details of the procedure. The user manual will consist of the details of how to access and use the special crack front elements and several sample problems. This user manual is being designed such that it will be self-contained and may be used without the technical report, if the user chooses to. Substantial portions of the technical report and user manual have been completed.

E-23-623

R & D Status Report

on

Bi-Material Fracture Program

under contract

AFRPL F04611-76-C-0078

K. Kathiresan
Post-Doctoral Fellow

Satya N. Atluri
Project Director

June 1978

The problem of corner crack at the cylinder-nozzle intersection was solved successfully. For the purpose of comparision of the solution of the variation of the stress-intensity factors along the crack front, the problem was choosen from an open literature where the solution was obtained by using a stress freezing photoelasticity technique coupled with digital computer. The result by the TEXGAP-3D-CRACK program gave excellent correlation with the experimental results. The result of this problem will be given in detail in the volume II of the technical Report. In this problem, the crack face was also pressurized along with the internal pressure in the cylinder and the nozzle. The problem of inner surface flaw in a pressurized cylinder with thermal loading was also solved successfully. The results of this problem will also be given in detail in the technical Report. Other problems of bi-material fracture, which we wanted to solve in Georgia Tech CDC-6600 have all been solved already.

A total number of seven problems, four of which on homogeneous fracture and the rest on bi-material fracture, will be presented in detail in Volume II of the technical report along with the user manual. The Volume I of the technical report which contains the theortical background of the hybrid displacement finite element model has been completed and is being typed. Volume II which contain the user manual and the sample problems is almost complete and the inputs and outputs of the sample problems are now being documented. This also will be typed immediately after Volume I.

E23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

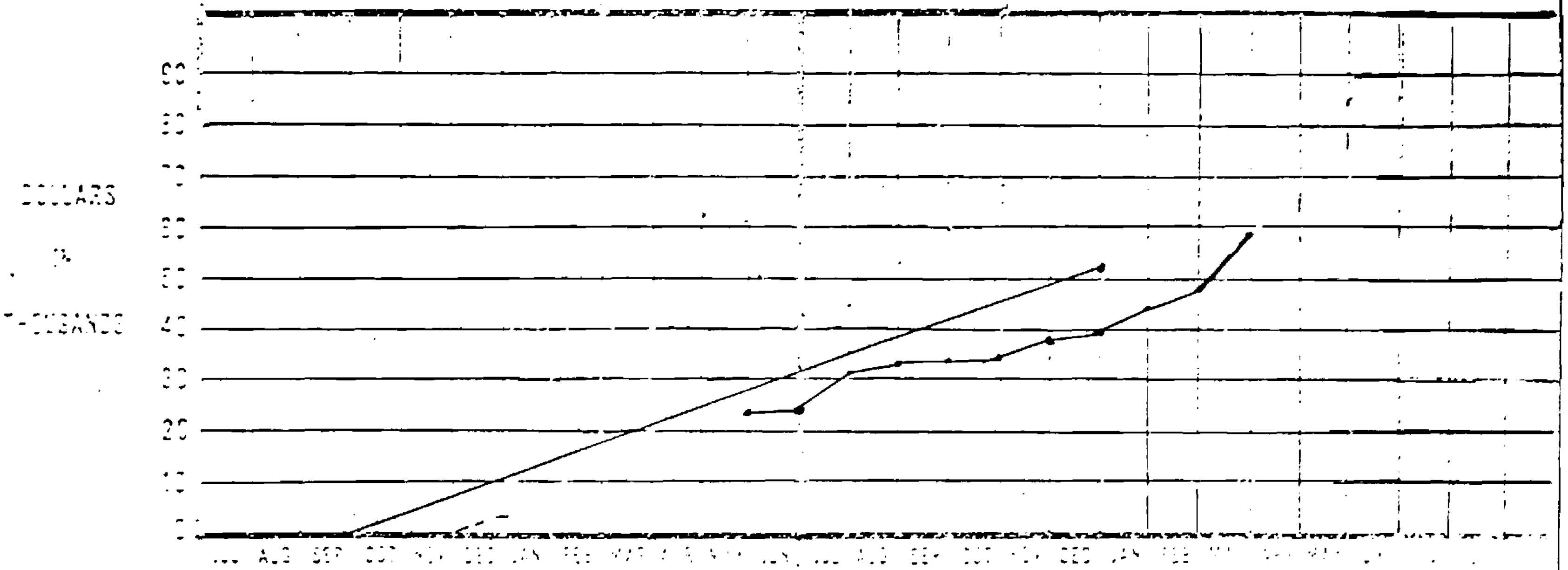
1 Copy

CONTRACT No. F04611-76-C-0078

REPORTING ORGANIZATION: Georgia Institute of Technology

AS OF DATE: March 31, 1978

SUBMISSION DATE: April 12, 1978



Planned Cost ————————

Actural Cost - - - - - - - - - - - - -

FY-77 FY-78

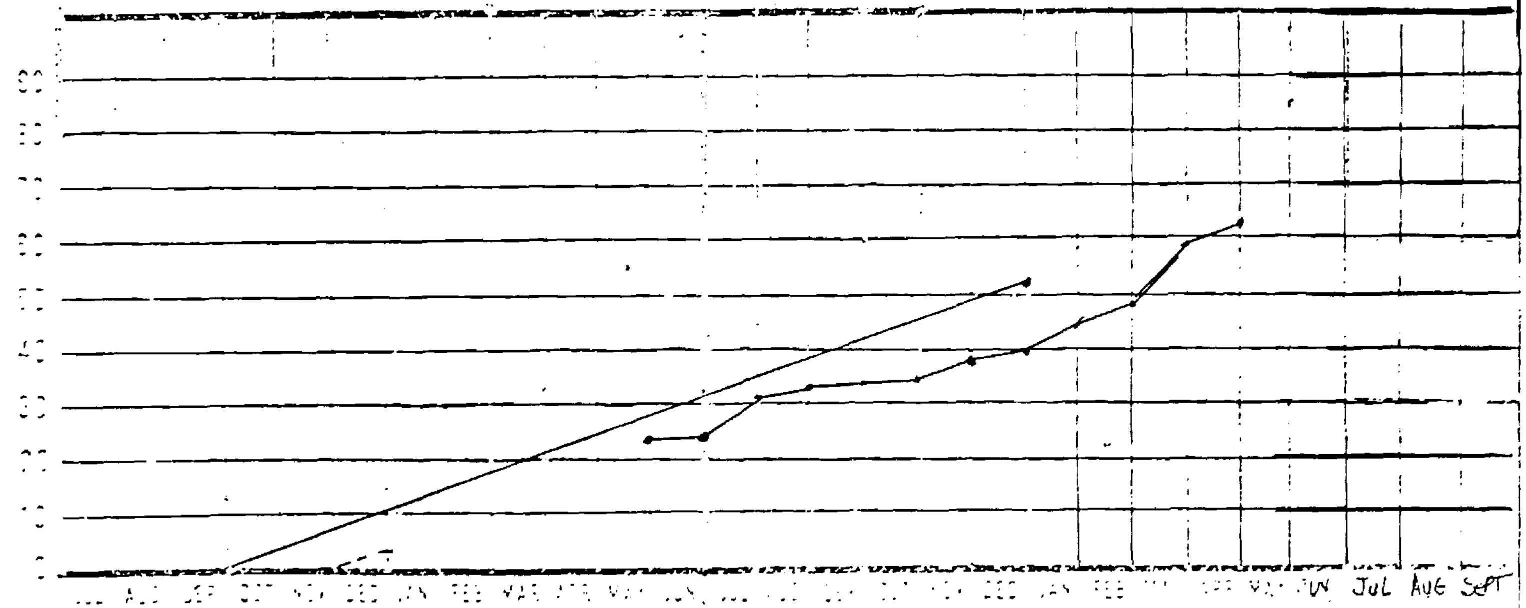
PLANNED COST	4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	[illegible]	[illegible]	[illegible]	84
ACTUAL COST	0	0	3	6	[illegible]	12	[illegible]	[illegible]	[illegible]	[illegible]	33	[illegible]	[illegible]	[illegible]	39	[illegible]	[illegible]	59	62
REVISED ESTIMATE																			

FINANCIAL FORECAST VS. ACTUAL REPORT

E-23-623

CONTRACT No.	REPORTING ORGANIZATION	
F04611-76-C-0078	Georgia Institute of Technology	AS OF DATE: April 30, 1978
		SUBMISSION DATE: Aug 7, 1978

DOLLARS IN THOUSANDS



Planned Cost ————————

Actual Cost - - - - - - - - - - - - -

FY-77 FY-78

PLANNED COST	4 7 10 14 18 21 25 28 31 35 39 42 46 49 50 [illegible] 74
ACTUAL COST	0 0 3 6 [illegible] 12 [illegible] 59 62
REVISED ESTIMATE	

FINANCIAL FORECAST VS. ACTUAL REPORT

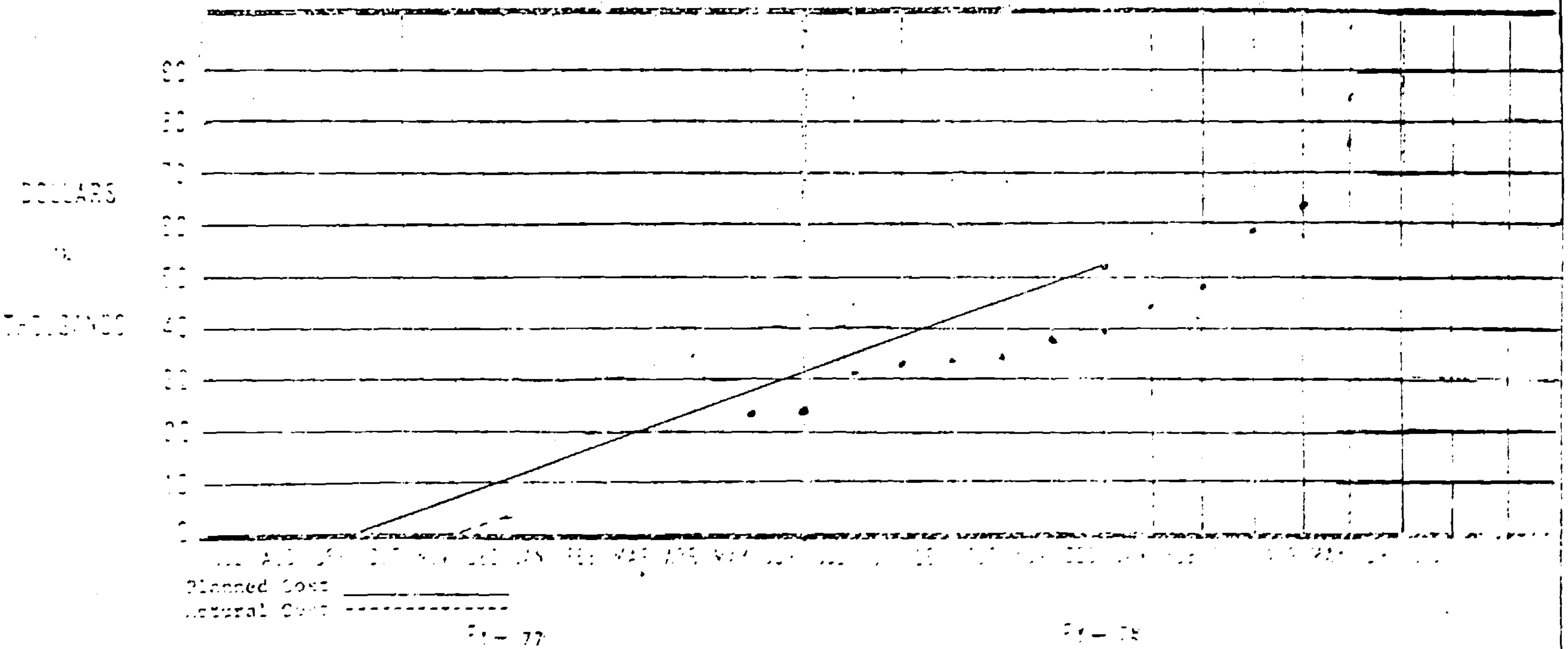
E-23-623

CONTRACT NO. F04611-76-C-0078

REPORTING ORGANIZATION: Georgia Institute of Technology

AS OF DATE: May 31, 1978

SUBMISSION DATE: Aug 7, 1978

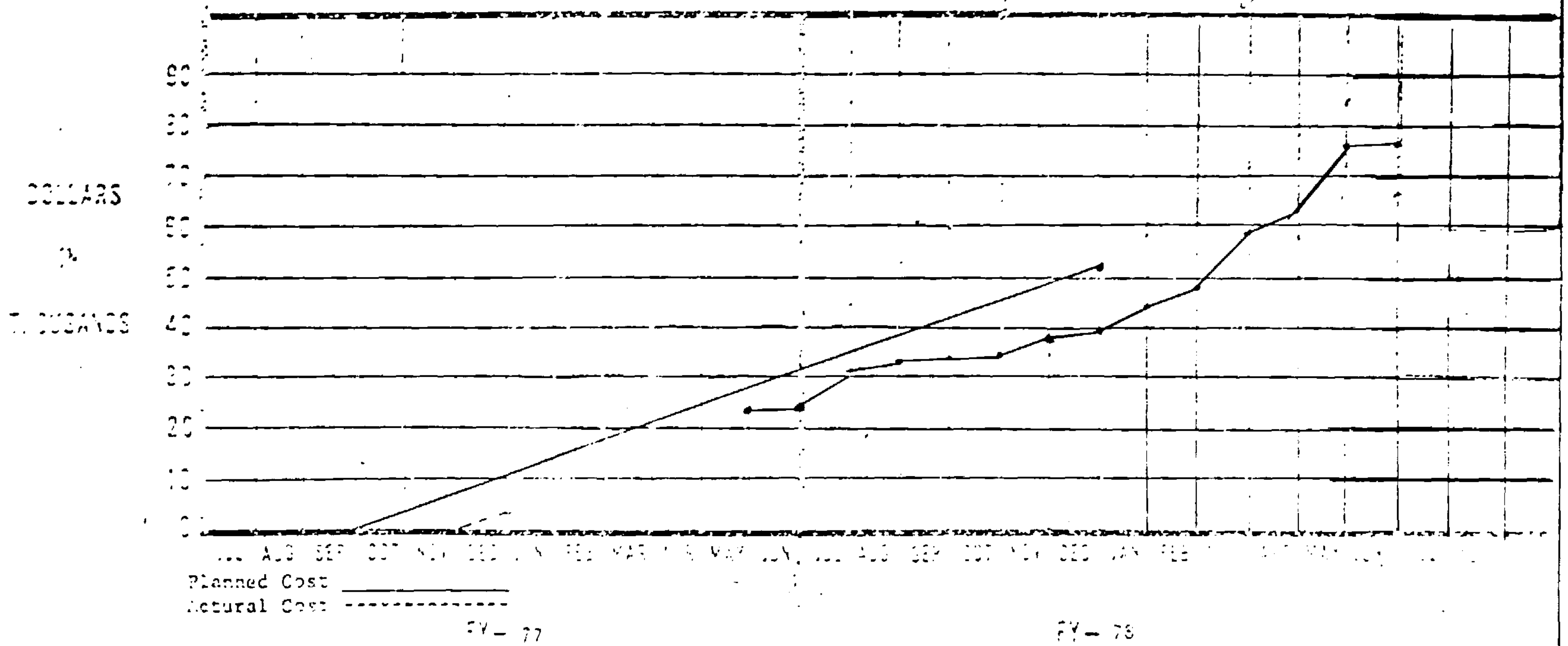


PLANNED COST	4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	[illegible]	[illegible]	[illegible]	[illegible]	74
ACTUAL COST	0	0	3	6	[illegible]	12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	59	62	77
REVISED ESTIMATE																				

FINANCIAL FORECAST VS. ACTUAL REPORT

E-23-623

CONTRACT No. F04611-76-C-0079	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE June 30, 1978 SUBMISSION DATE Aug 7, 1978



PLANNED COST	4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	74
ACTUAL COST	0	0	3	6	[illegible]	12	20	[illegible]	23	[illegible]	33	33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	59	62	77	
REVISED ESTIMATE																					

FINANCIAL FORECAST VS. ACTUAL REPORT

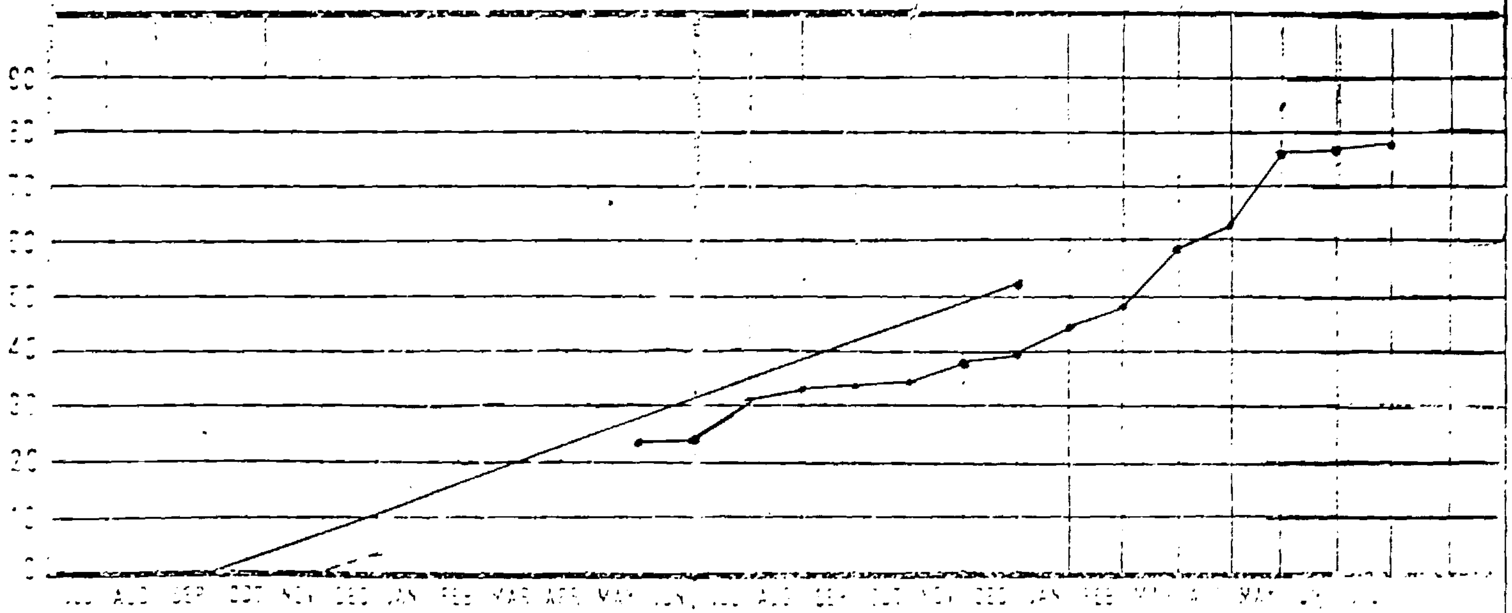
E-23-623

CONTRACT No. F04611-76-C-0078

REPORTING ORGANIZATION Georgia Institute of Technology

AS OF DATE July 31 1978

SUBMISSION DATE Aug 9, 1978



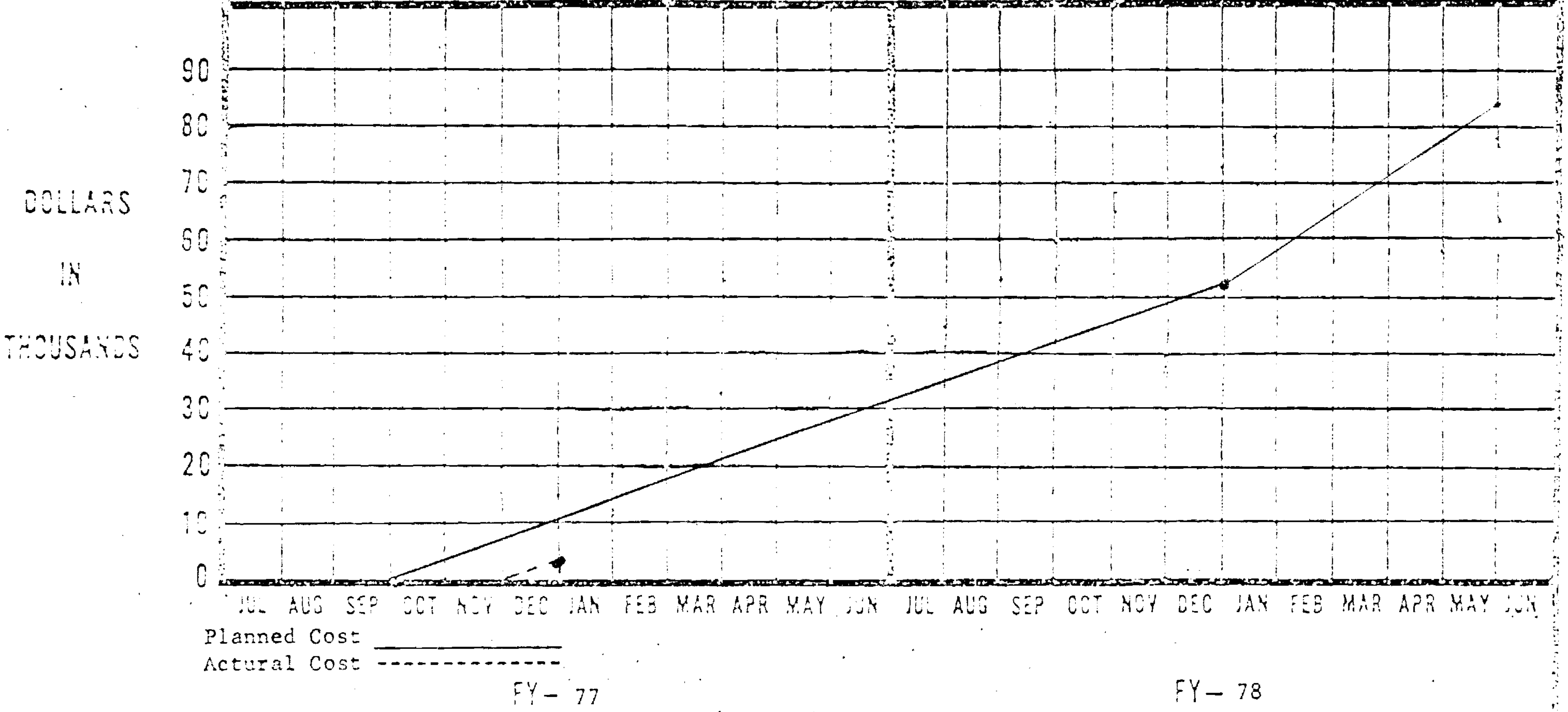
FY- 77 FY- 78

PLANNED COST	4 7 10 14 18 21 25 28 31 35 39 42 46 49 50 [illegible] 74
ACTUAL COST	0 0 3 6 [illegible] 59 62 77 78
REVISED ESTIMATE	

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE Dec 1976 SUBMISSION DATE

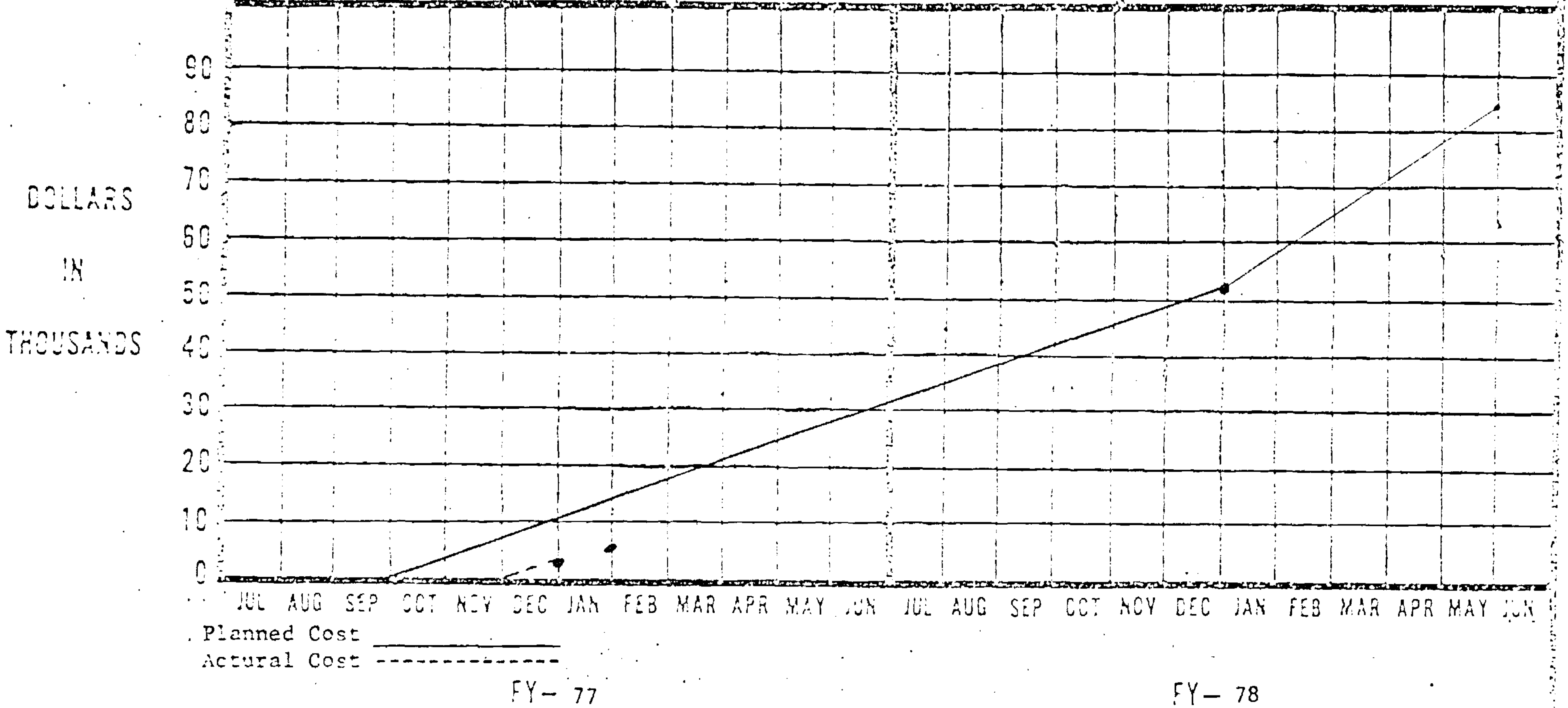


	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3																		
REVISED ESTIMATE																								

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE Jan 1977 SUBMISSION DATE

E-23-623

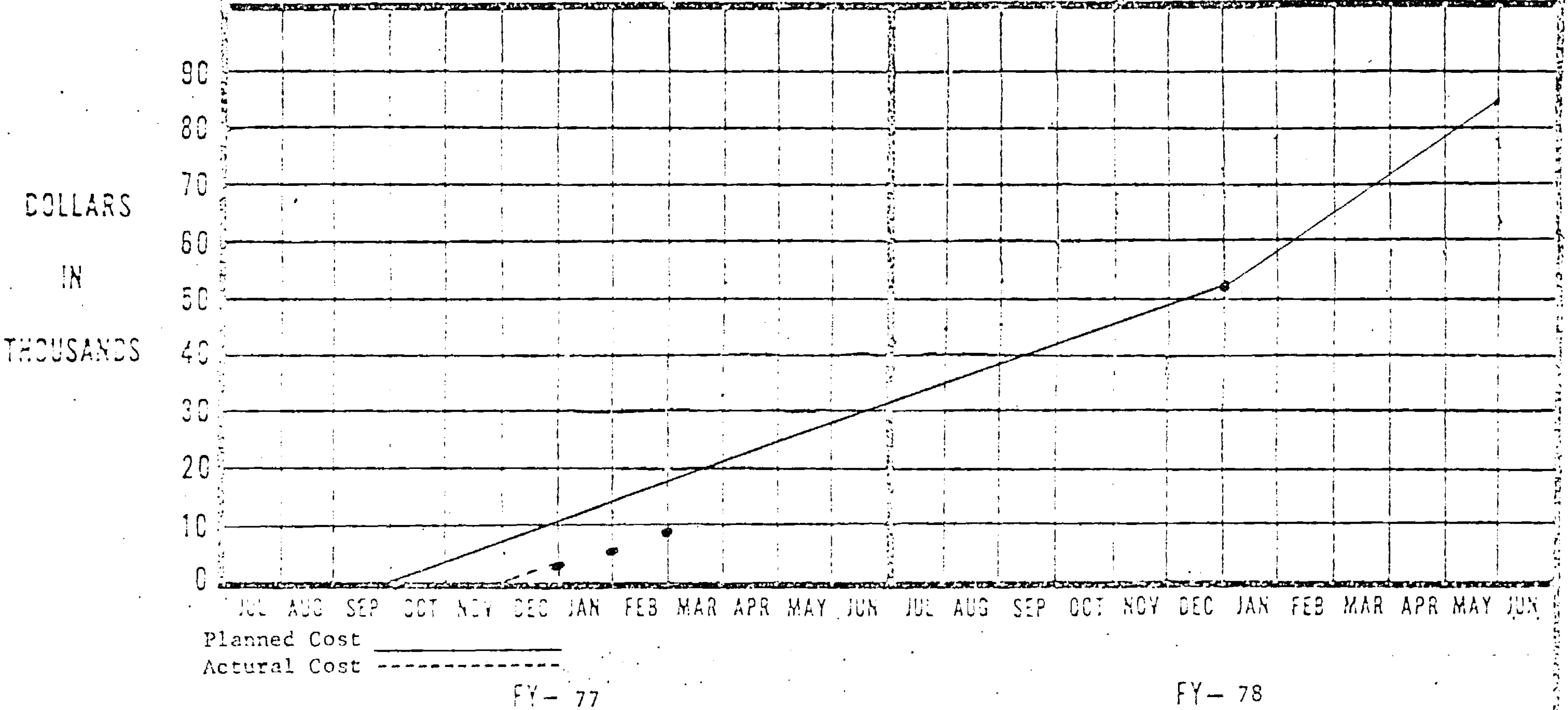


	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6																	
REVISED ESTIMATE																								

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE Feb 1977 SUBMISSION DATE

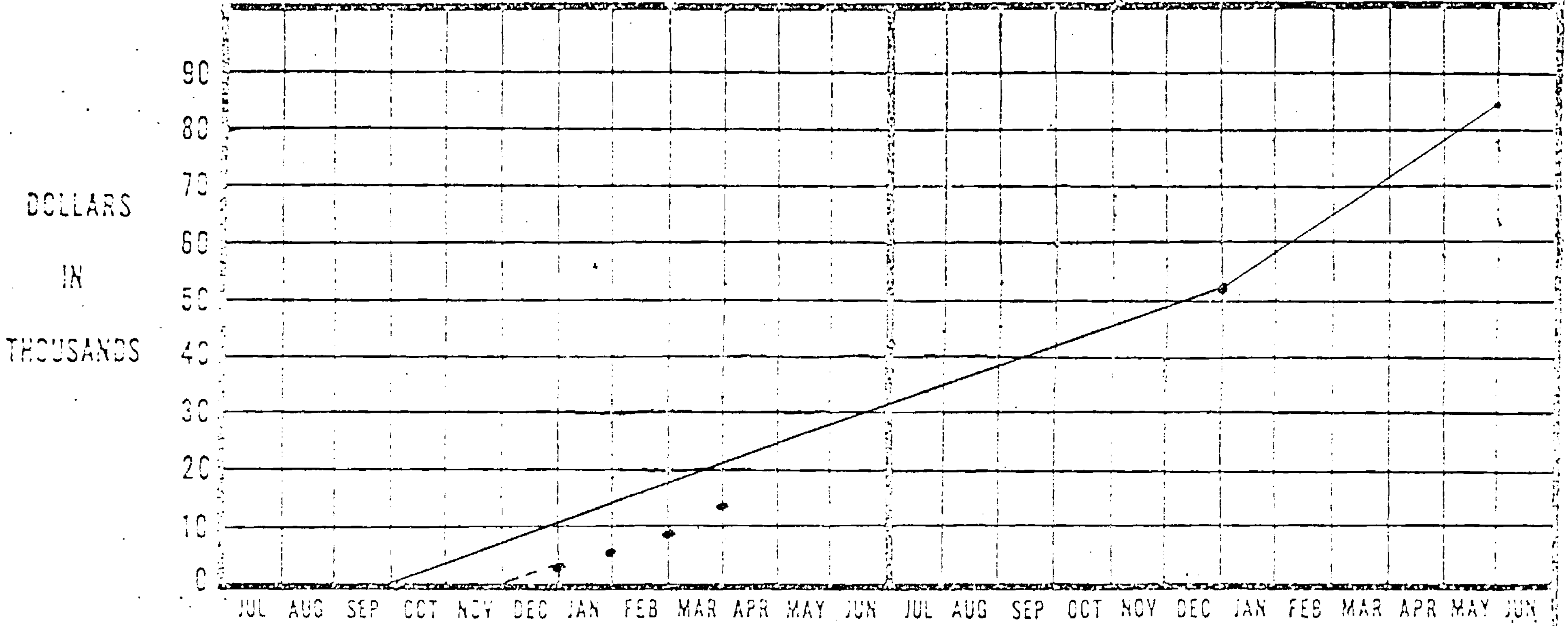


	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9																
REVISED ESTIMATE																								

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION: Georgia Institute of Technology	AS OF DATE March 1977 SUBMISSION DATE



Planned Cost ______________
Actural Cost --------------

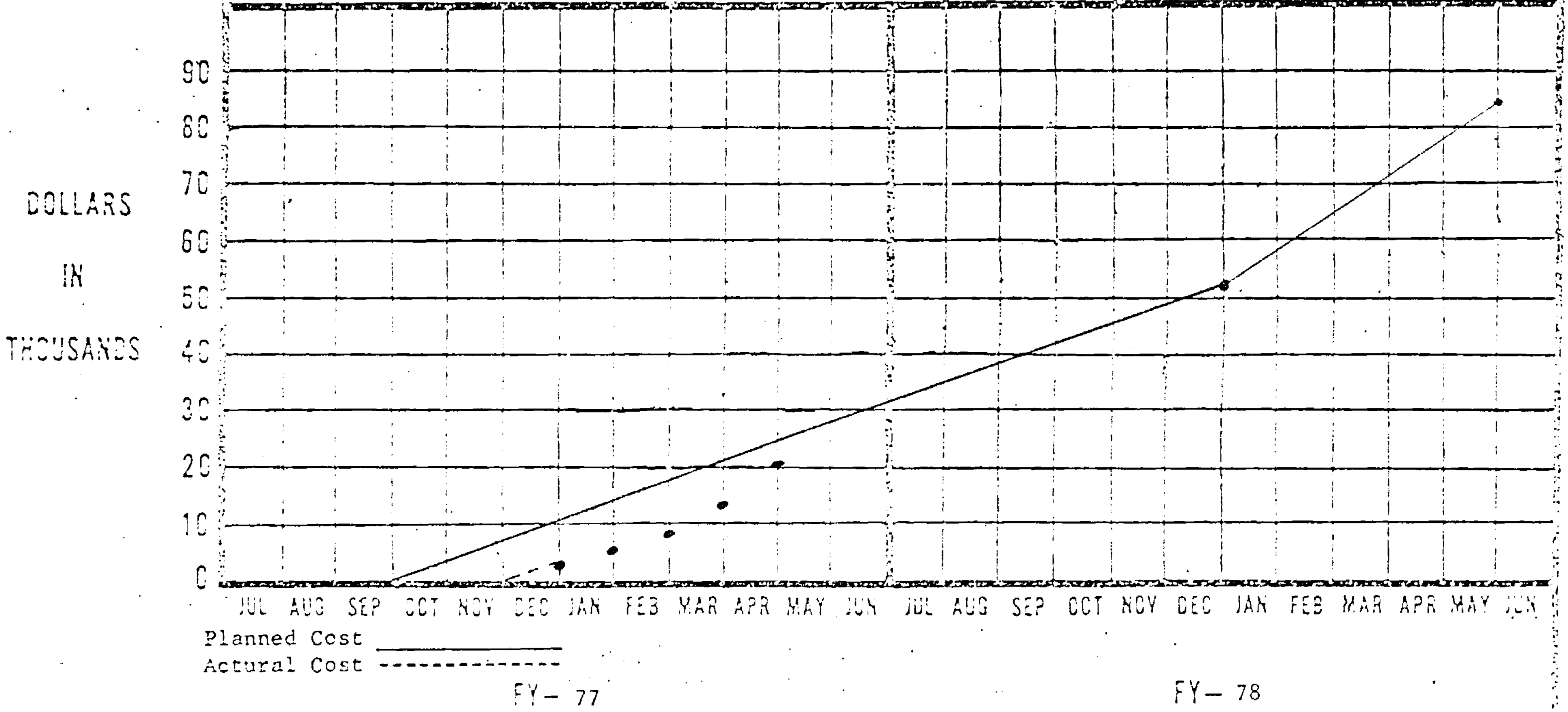
FY- 77 FY- 78

	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12															
REVISED ESTIMATE																								

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE April 1977 SUBMISSION DATE



	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20														
REVISED ESTIMATE																								

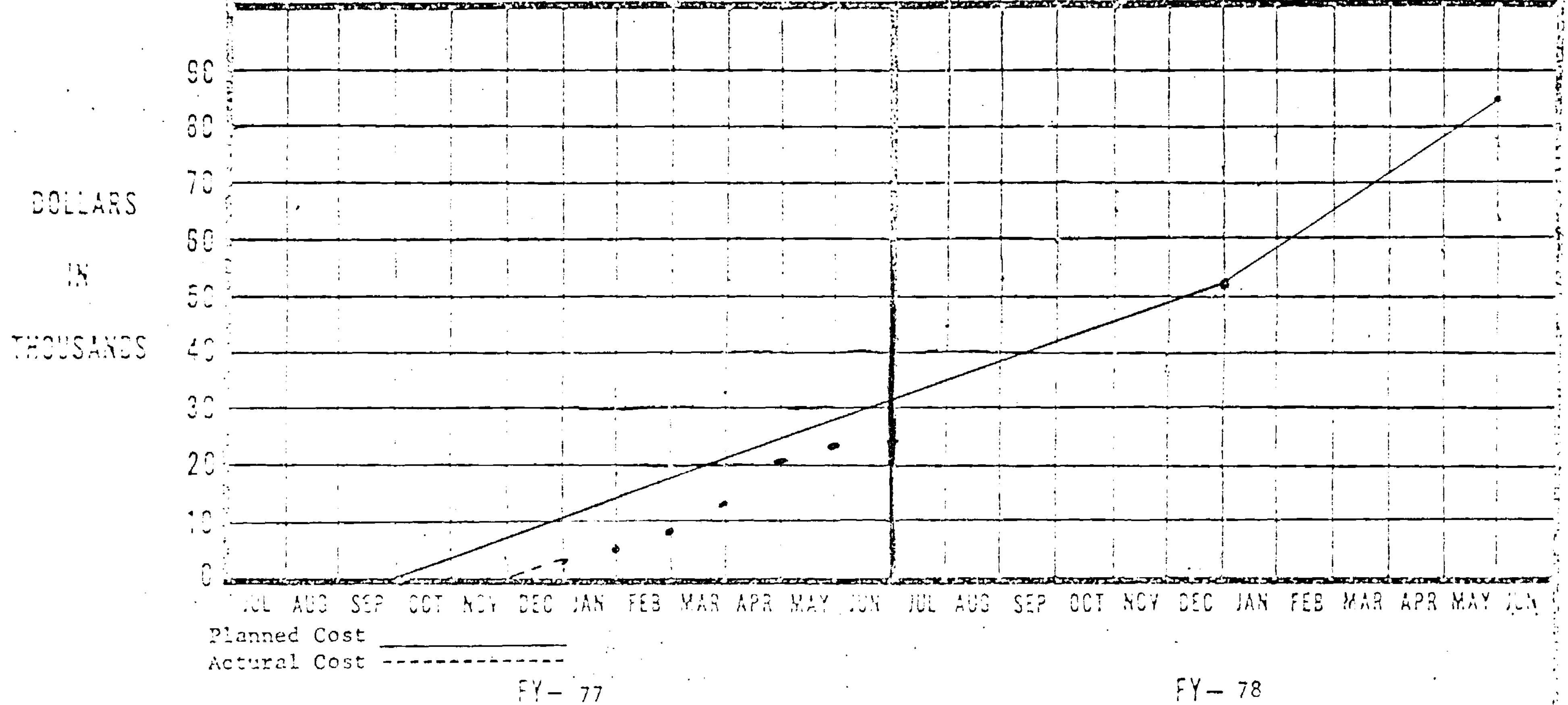
E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078

REPORTING ORGANIZATION
Georgia Institute of Technology

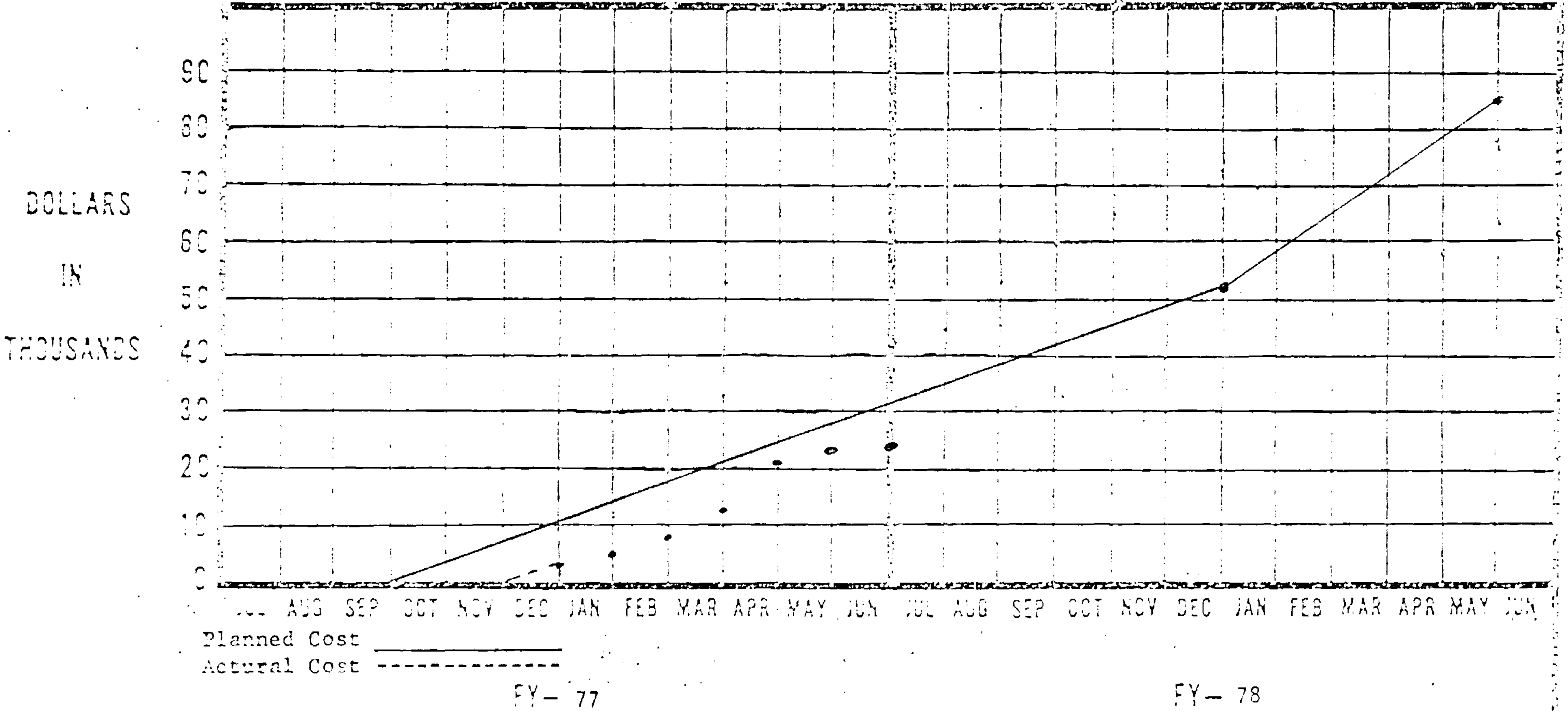
AS OF DATE May 1, 1977
SUBMISSION DATE



PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23													
REVISED ESTIMATE																								

E-23-623

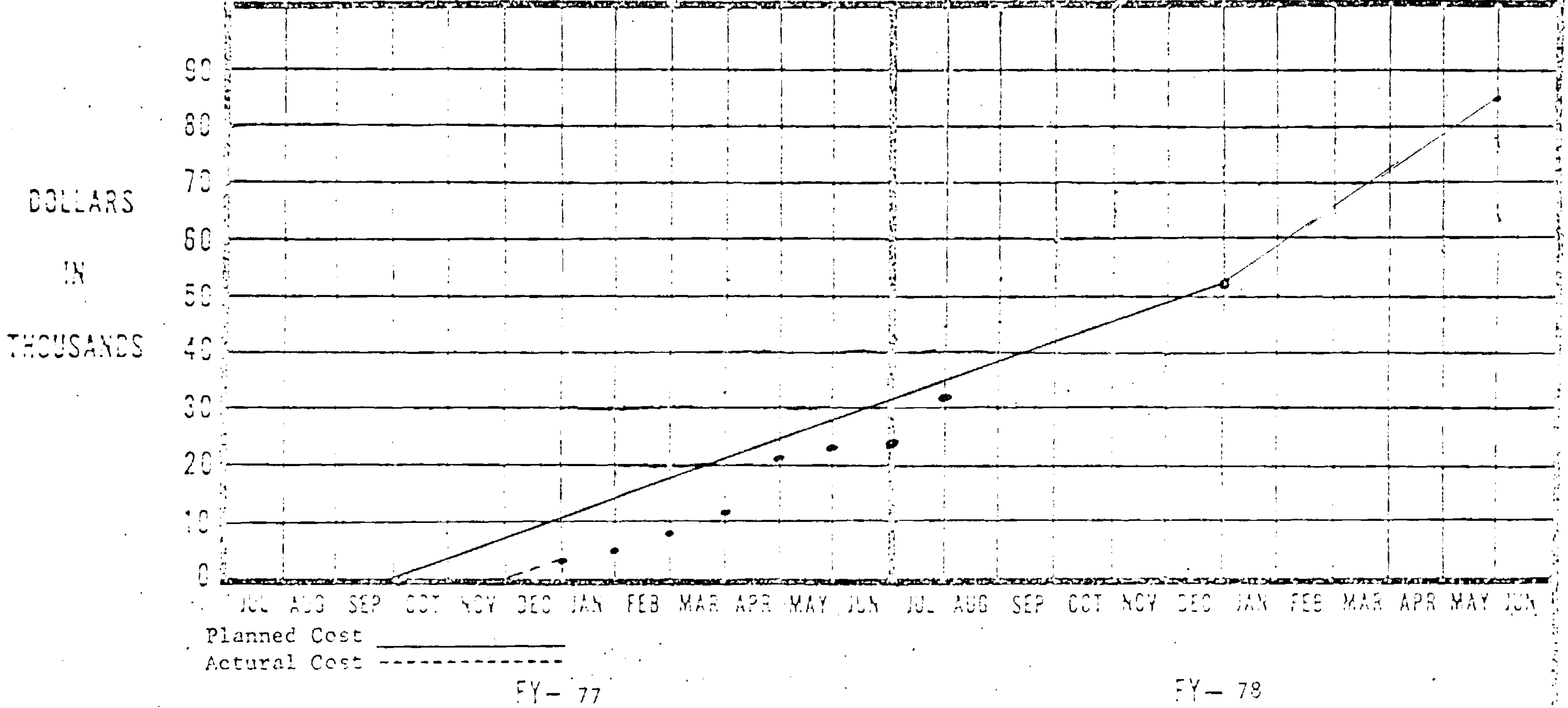
CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE June 1, 1977 SUBMISSION DATE



	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23												
REVISED ESTIMATE																								

E-23-623

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE July 1, 1977 SUBMISSION DATE



	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30											
REVISED ESTIMATE																								

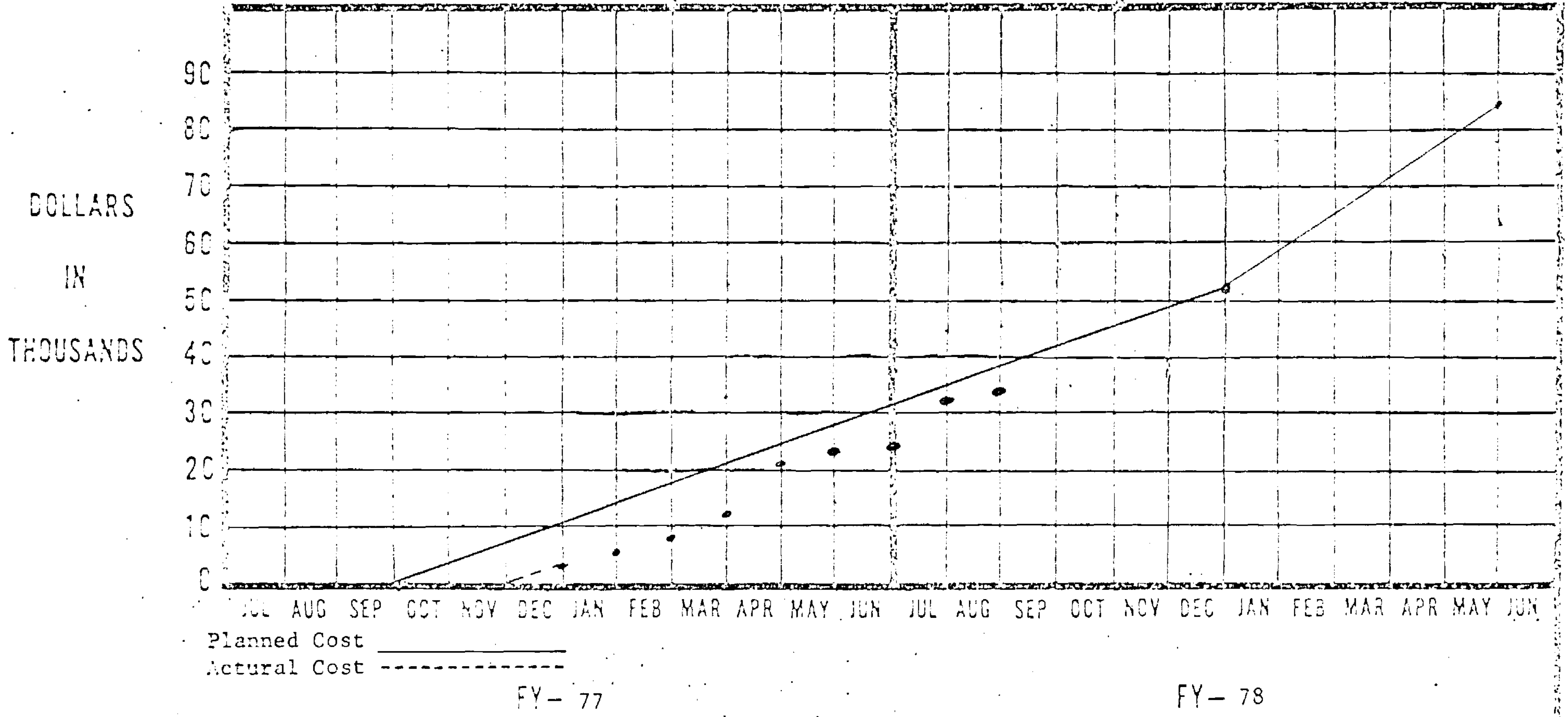
E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078

REPORTING ORGANIZATION
Georgia Institute of Technology

AS OF DATE Aug 1, 1977
SUBMISSION DATE

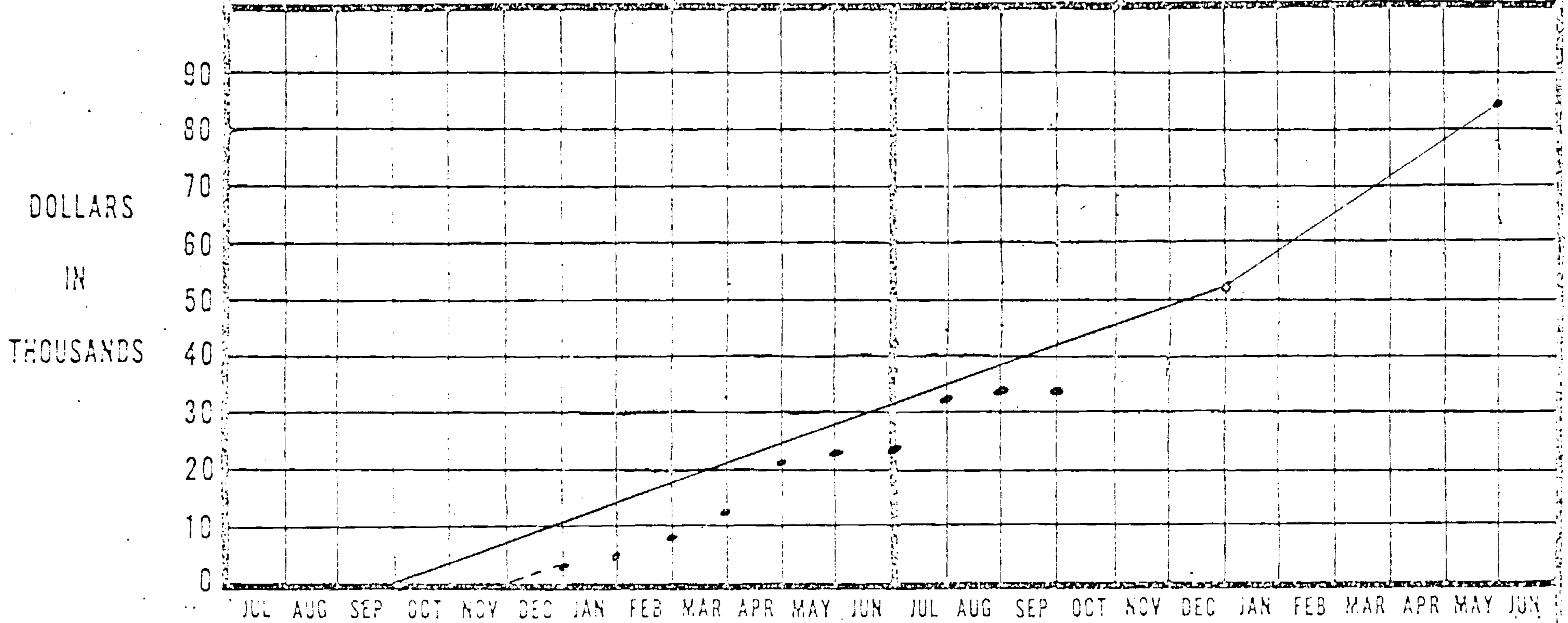


	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	55	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30	33										
REVISED ESTIMATE																								

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE Sept. 1, 1977 SUBMISSION DATE



Planned Cost ____________

Actural Cost - - - - - - - - - - - - -

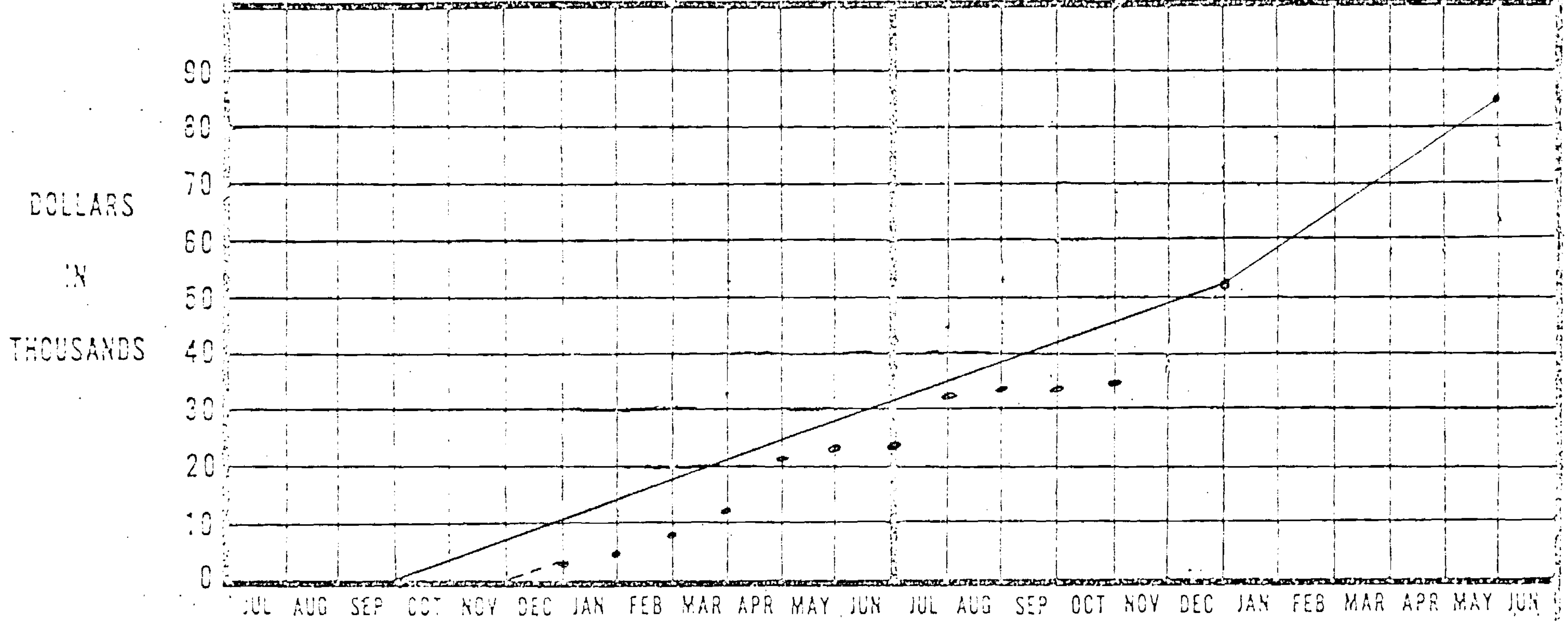
FY- 77 FY- 78

	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30	33	33									
REVISED ESTIMATE																								

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION: Georgia Institute of Technology	AS OF DATE Oct. 1, 1977 / SUBMISSION DATE



Planned Cost ______________

Actural Cost --------------

FY- 77

	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31
ACTUAL COST				0	0	3	6	9	12	20	23	23
REVISED ESTIMATE												

FY- 78

	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST	30	33	33	34								
REVISED ESTIMATE												

E-23-623

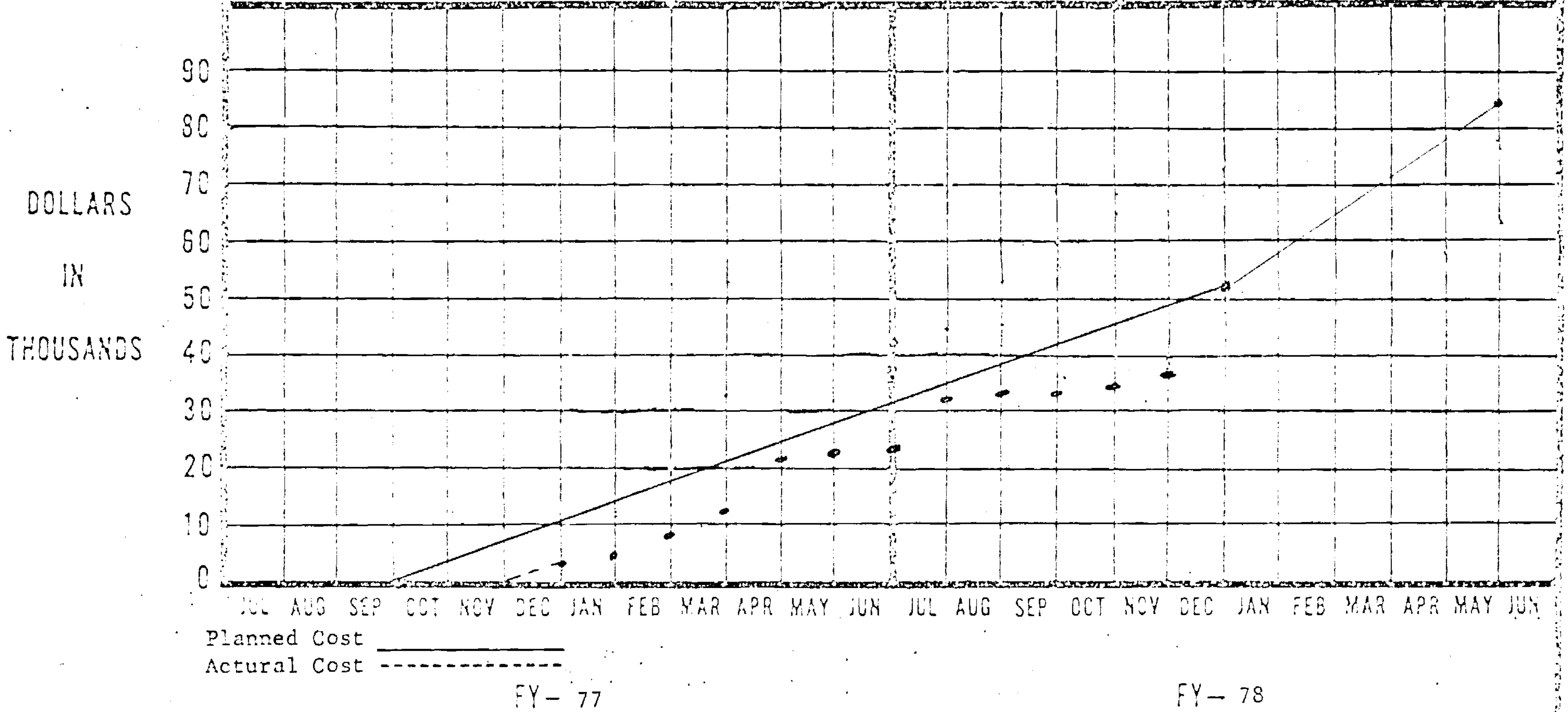
FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078

REPORTING ORGANIZATION: Georgia Institute of Technology

AS OF DATE: Nov 1, 1977

SUBMISSION DATE: ____________

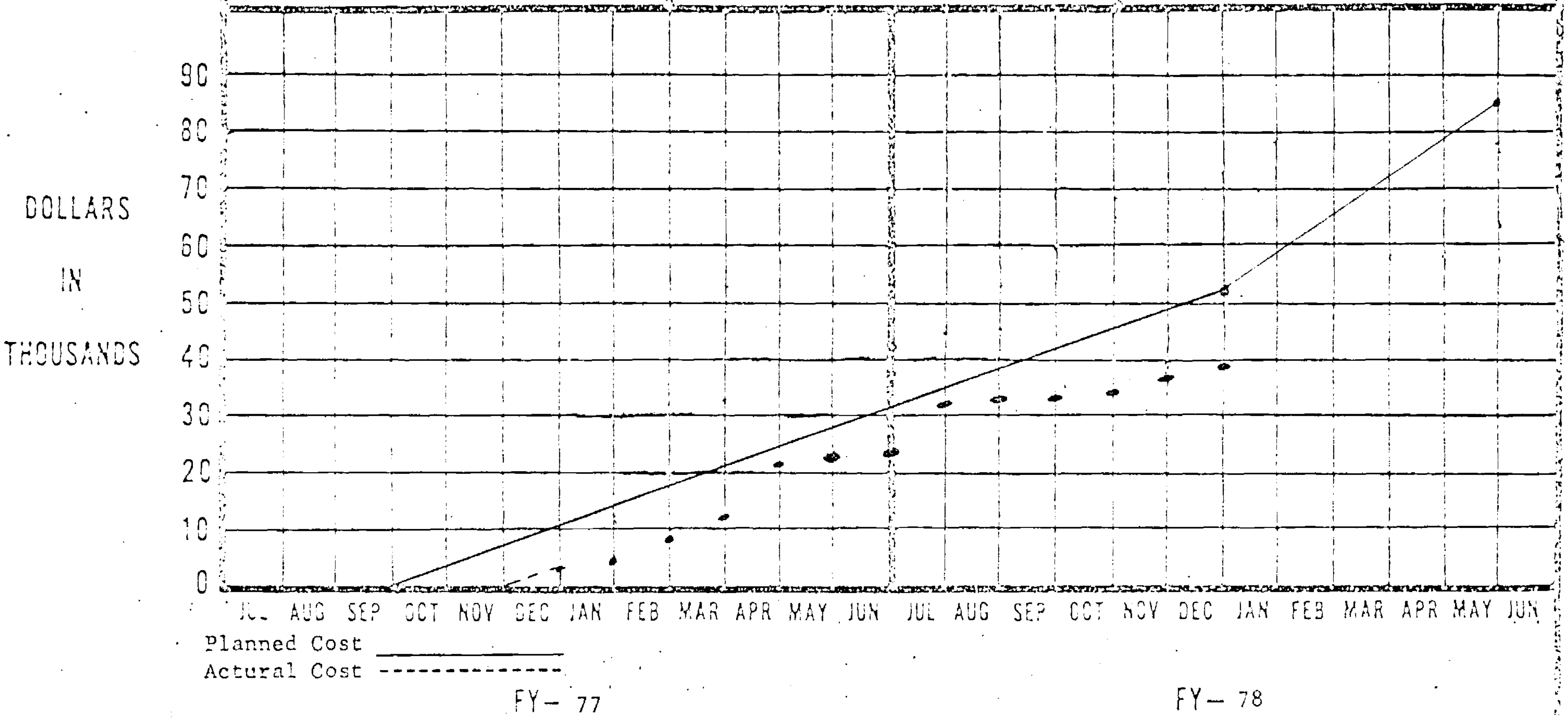


	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	55	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30	33	33	34	34							
REVISED ESTIMATE																								

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No.	REPORTING ORGANIZATION	AS OF DATE
F04611-76-C-0078	Georgia Institute of Technology	Dec. 1, 1977
		SUBMISSION DATE

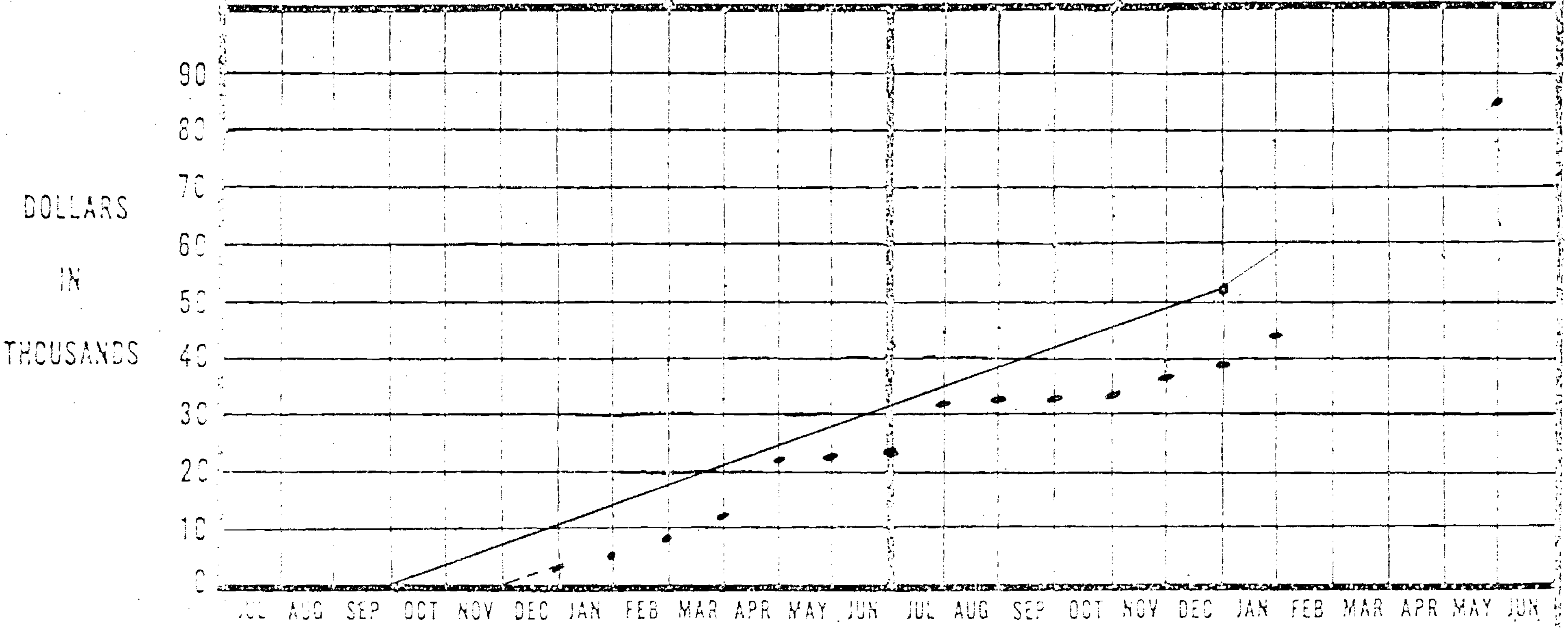


	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30	33	33	34	34	34						
REVISED ESTIMATE																								

E-23-623

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE Jan 1, 1978 SUBMISSION DATE



Planned Cost ______________
Actural Cost --------------

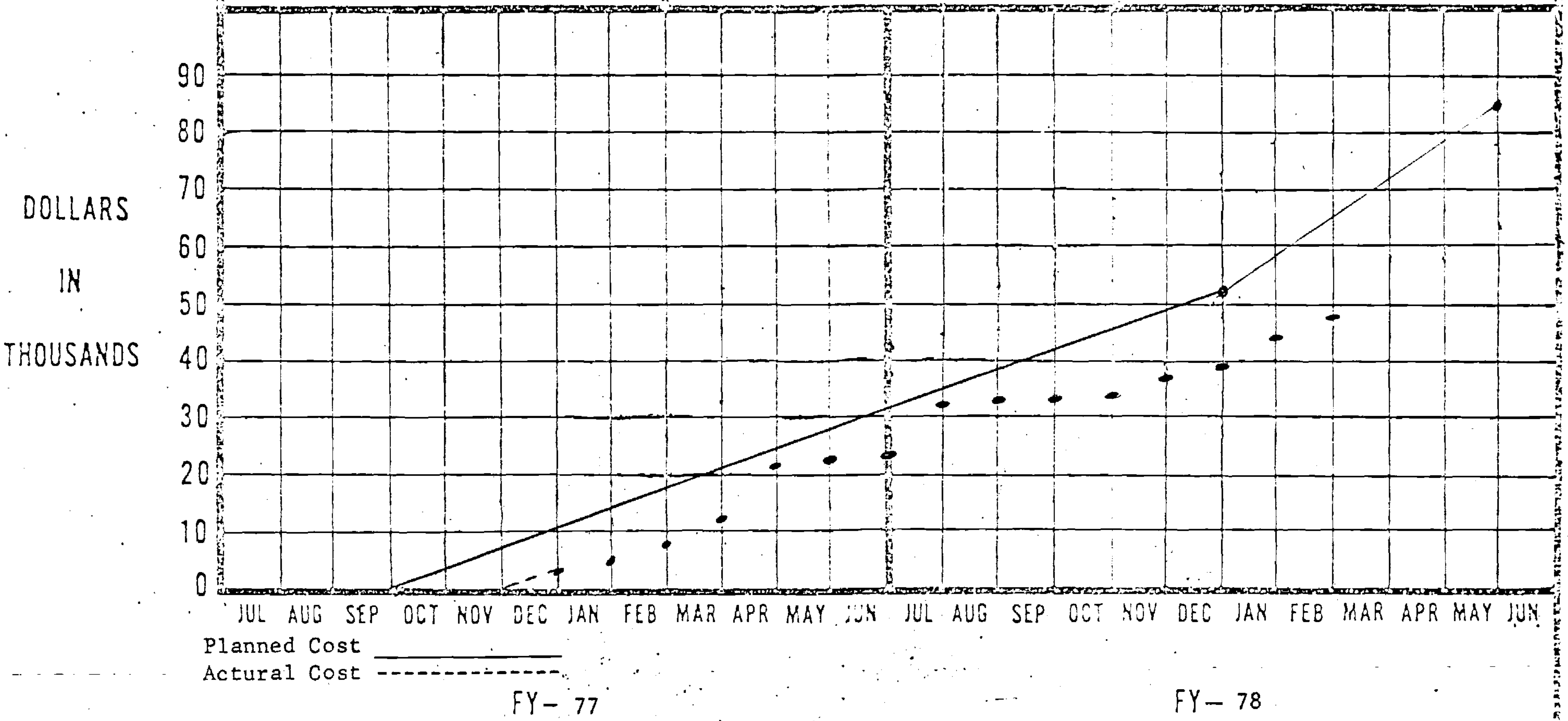
FY- 77 FY- 78

PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30	33	33	34	34	34						
REVISED ESTIMATE																								

E-23-622

FINANCIAL FORECAST VS. ACTUAL REPORT

CONTRACT No. F04611-76-C-0078	REPORTING ORGANIZATION Georgia Institute of Technology	AS OF DATE Feb 1, 1978 SUBMISSION DATE



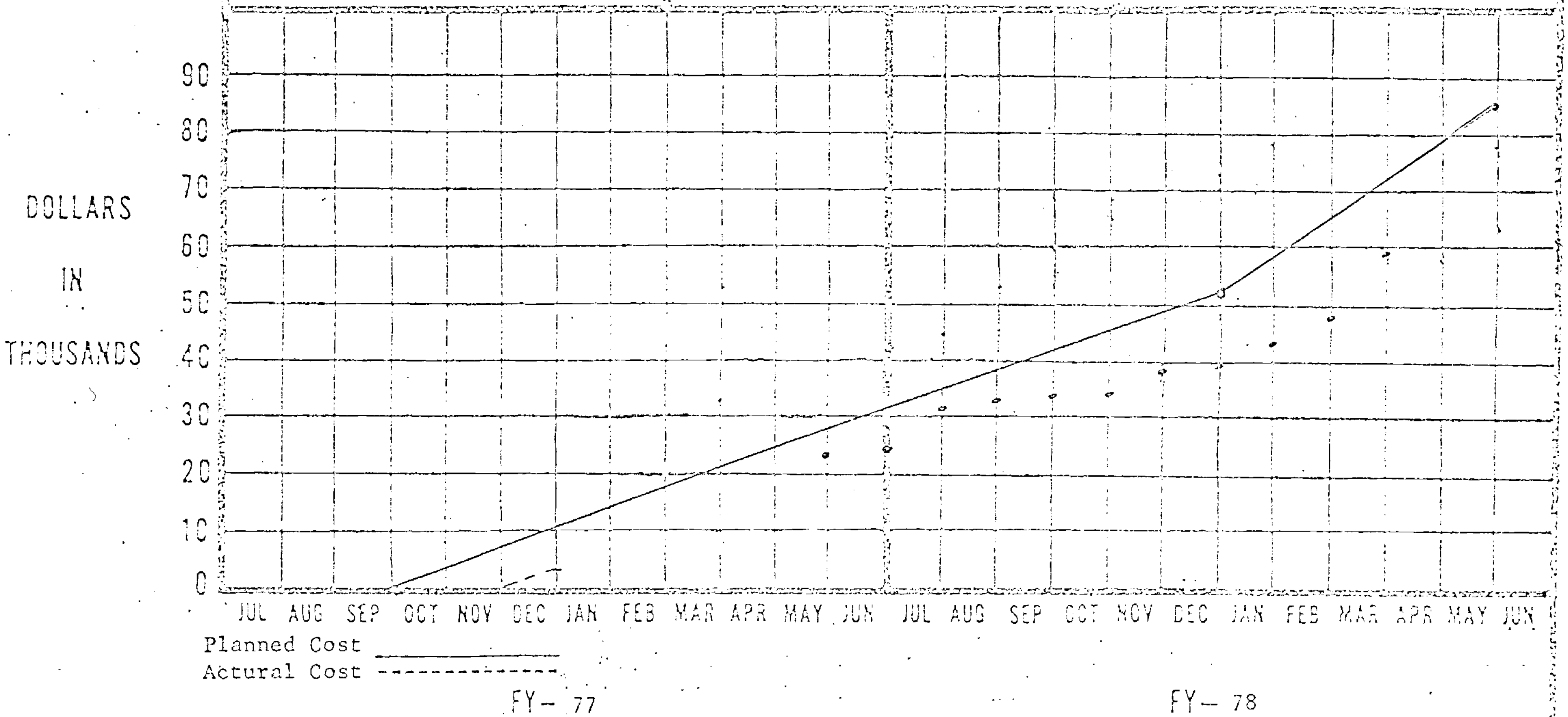
	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30	33	33	34	34	34	44					
REVISED ESTIMATE																								

FINANCIAL FORECAST VS. ACTUAL REPORT

E-23-623

E-23-623

CONTRACT No.	REPORTING ORGANIZATION	AS OF DATE / SUBMISSION DATE
F04611-76-C-0078	Georgia Institute of Technology	AS OF DATE 3-31-78 SUBMISSION DATE 4-19-78



	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
PLANNED COST				4	7	10	14	18	21	25	28	31	35	39	42	46	49	50	58	65	71	78	84	
ACTUAL COST				0	0	3	6	9	12	20	23	23	30	33	33	34	34	34	44	48	59			
REVISED ESTIMATE																								

xc: Al Becker

E 23-623

Progress Report

on

Bi-Material Fracture Program

under contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan, Post-Doctoral Fellow

S.N. Atluri, Project Director

Progress Report

on

Bi-Material Fracture Program

under contract

AFRPL F04611-76-C-0078

Submitted by

K. Kathiresan, Post-Doctoral Fellow

S.N. Atluri, Project Director

Attention was focused in the beginning to carry out the following tasks as the initial phase of the contract. (1) Loading of the TEXGAP-3D computer program in CDC Cyber 74 Computer at Georgia Institute of Technology. (2) Modifying the three-dimensional homogeneous cracked element computer program, developed at Georgia Tech, such that the program is compatible and can be easily incorporated into TEXGAP-3D coding. This homogeneous cracked element was primarily developed under AFOSR grant 74-2667, as a theoretical research tool and thus didn't involve optimized computer time and memory. Thus the homogeneous cracked element program has been modified for optimization in computer time and memory. (3) Extensive literature on two and three-dimensional bi-material fracture problems such that the computer program which will be developed would be as general as possible to solve as many different kinds of problems of bi-material fracture as possible.

The TEXGAP-3D program has been already installed in CDC Cyber 74 computer of Georgia Institute of Technology. Trial problems for which the solutions are already available (which were provided by Thiokol/Huntsville Division) were run, before incorporating the three-dimensional homogeneous cracked element program, so as to check the overall installation of TEXGAP-3D program. The effort was very successful and produced results identical to those provided by Thiokol/Huntsville Division. In the modification of three-dimensional homogeneous cracked element computer program, efforts were focused on the following, so that it can be incorporated successfully in TEXGAP-3D computer program: (a) To reduce the size of the homogeneous cracked element program to be as small as possible by removing all unnecessary operations such as printing the stiffness matrix, computing and checking the residual(unequilibrated)nodal forces of the stiffness matrices, assuming hypothetical values of stress-intensity factors and computing them back using the computed element properties, etc. (b) to reduce the total memory storage used by homogeneous cracked element program so that overlaying of FORMK3 in TEXGAP-3D along with the added cracked element program can be done easily without increasing the total memory required by TEXGAP-3D program. (c) to optimize the cracked element program as much as possible so that the compilation time and execution time for developing the stiffness matrix of a singular element will be reduced. The above three tasks have been completed successfully. For the homogeneous cracked element program, computer memory and execution time have been reduced by half and one third respectively.

After modifying and optimizing the homogeneous cracked element, it was incorporated in overlay FORMK3 of TEXGAP-3D. In the version of TEXGAP-3D provided to us, there were totally nine elements namely BRICKH, BRICK, PRISMH, PRISM, TETRAH, TETRA, DEGENH, DEGEN, and TIEELM. The four types of homogeneous cracked elements are now placed after the ninth TIEELM element. Extensive modifications in the overlays SETUP and FORMK3 were made to include the four cracked elements. The node numbering of the element in the original cracked element program and TEXGAP-3D program were different. The cracked element program is now modified to match the node numbering of TEXGAP-3D program. The program now calculates the stiffness matrices of the singular elements and places (upper or lower) triangular portion of the stiffness matrices properly in the dimensioned variable STRAN in overlay FORMK3. The stiffness matrices of singular elements generated by the cracked element program, that is now merged in TEXGAP-3D program, have been printed out and checked for their accuracy.

The TEXGAP-3D program does not have the facility to copy the stiffness matrix of a regular elements if the regular elements are exactly identical.

But such a facility has been incorporated already for cracked singular elements program. This will result in saving of computer time. This was done, essentially, by the use of a tape. Before starting to compute the stiffness matrix of a singular element, the program first checks whether the present element is identical to any of the previously computed elements. If so, it simply duplicates the stiffness matrix. This is one of very essential and important features to save computer time.

Then the overlay STRESS was modified to obtain the stress-intensity factors corresponding to those singular cracked elements. Extensive modifications were made in this overlay also. The program now computes the stress-intensity factors for singular elements instead of stresses. The program now prints the displacements of elements first, then if it is a regular element it computes the stresses; and if it is a singular element, it computes the stress-intensity factors. This facility is incorporated in BLOCK option of the STRESS overlay.

All the above mentioned tasks have been accomplished successfully. Now the program is being modified to accommodate other conditions such as arbitrary pressure distribution on the crack surface and thermal effects in homogeneous cracked element. These changes would be accomplished in the near future.

With the point of view of making the three-dimensional bi-material computer program, which will be developed, as general as possible, extensive literature search has been done on existing methods and solutions for two as well as three-dimensional bi-material fracture problems. The stress distribution near the crack in a plane strain bi-material fracture problems are given in two different forms in the literature. The stresses were rederived to check whether these two stress distributions are the same. It was found that even though the procedure used is the same in both cases the stress functions used to derive the stress-distribution were different. Using these stress-distributions near the crack for a bi-material fracture problems, the three field variables namely, (1) the arbitrary interior displacements for the element, (2) inter-element boundary displacements, and (3) the element boundary tractions are being developed. This task also would be completed in the near future and the computer coding for three-dimensional bi-material cracked element program would be initiated.

E-23-623

Program Plan

Bi-Material Fracture Program

contract F04611-76-C-0078

Approved by

Satya N. Atluri, Project Director

M. E. Raville, Director, School of ESM

April 1977

Program Plan

Bi-Material Fracture Program

contract F04611-76-C-0078

Approved by

Satya N. Atluri, Project Director

M. E. Raville, Director, School of ESM

April 1977

1. Introduction

This program plan describes work, to be accomplished, which will result in incorporation of both homogeneous and bi-material cracked finite elements into the user oriented three-dimensional finite element code TEXGAP-3D. AFRPL contract F04611-76-C-0078 authorizes work to be conducted.

2. Program Summary

The primary objective of this research program is to develop a displacement-hybrid finite element procedure capable of treating both homogeneous and bi-material fracture mechanics calculations in 3-dimensions. The homogeneous cracked finite element developed under AFOSR Grant 74-2667 at Georgia Institute of Technology, under the direction of S. N. Atluri, and the planned development for the bi-material cracked finite element will be incorporated into the existing TEXGAP-3D computer code; thus providing the capability for fracture-mechanics based structural integrity assessment of solid propellant rocket motors. In addition, both types of cracked-element formulations will be enhanced to include thermal effects and effects of arbitrary crack-face pressure distributions.

This program will be composed of three tasks. In the first task, the contractor will extensively modify the existing 3-dimensional homogeneous material cracked element to provide the desired user-oriented engineering analysis tool of a bi-material cracked finite element. The second task will incorporate both homogeneous and bi-material cracked finite elements into TEXGAP-3D. In the third task, the code will be verified through the evaluation of homogeneous and bi-material crack problems for which solutions are known. In addition, a detailed user's manual will be produced with approaches on how to set up, model, solve, and assess the results of typical homogeneous and bi-material crack problems which are relevant to the solid propellant industry.

3. Schedule

The master program schedule is given in the following figures. All technical effort will be completed in 16 months followed by a 3-month period for preparation and approval of the final report.

Program Schedule

Contract Award

Phase I. Bi-Material Cracked Element Development

Task 1. Development of asymptotically correct element-interior displacement field for bi-material element.

Task 2. Development of asymptotically correct boundary-displacement field for bi-material element.

Task 3. Development of asymptotically correct boundary traction field for bi-material element.

Task 4. Computer coding for obtaining condensed stiffness matrix for bi-material crack element.

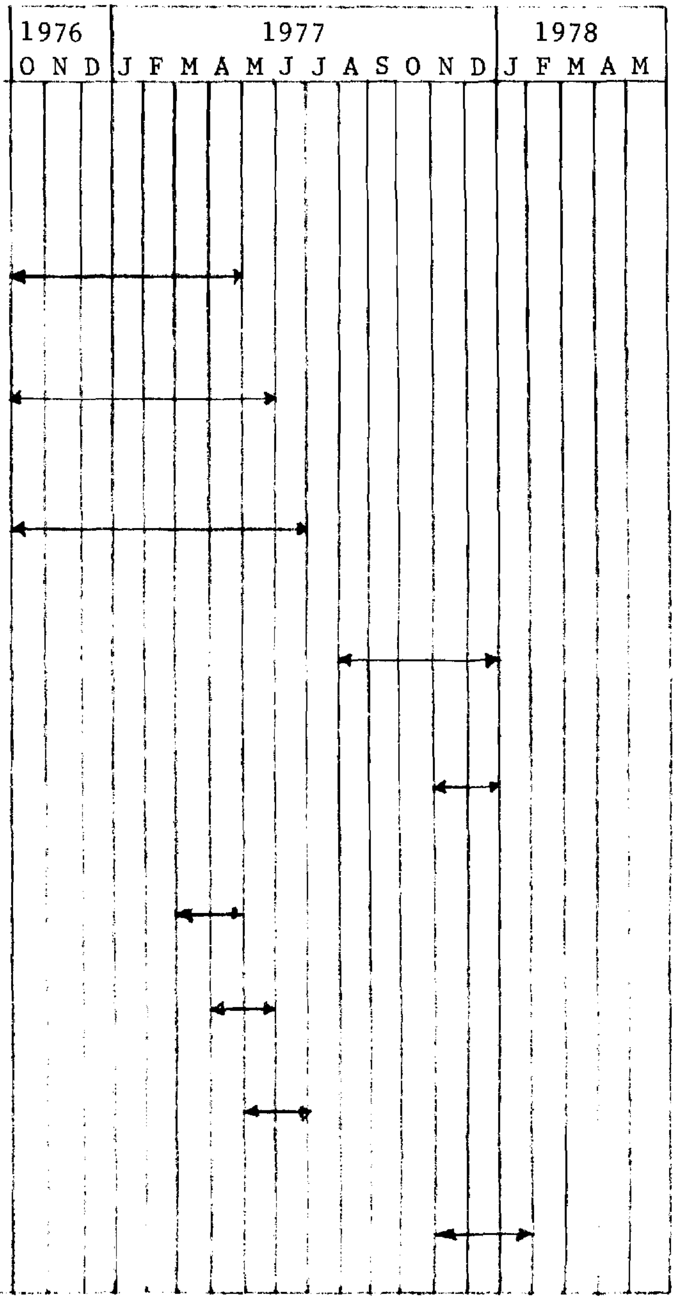
Task 5. Verification of properties of bi-material crack element stiffness matrix.

Task 6. Incorporation of thermal effects in homogeneous crack element.

Task 7. Incorporation of crack-face pressure for homogeneous element.

Task 8. Checking compatibility of developments in tasks 6 and 7 with TEXGAP-3D.

Task 9. Repetition of tasks 6, 7, and 8 for bi-material crack element.



	1976			1977												1978								
	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J	F	M	A	M	J	J	A	S
Phase II. Incorporation of Cracked Elements in TEXGAP-3D																								
Task 1. Optimization of coding for existing homogeneous element.					—	—	—	—																
Task 2. Incorporation of homogeneous element (without thermal and pressure effects) into TEXGAP-3D.	—	—	—	—	—	—	—																	
Task 3. Incorporation of homogeneous element with thermal and pressure effect capability into TEXGAP-3D.									—	—														
Task 4. Incorporation of bi-material element with thermal and pressure effect capabilities into TEXGAP-3D.														—	—	—								
Phase III. Verification																								
Task 1. Verification problems for homogeneous element (with thermal and pressure effects).											—	—	—	—	—									
Task 2. Verification problems for bi-material element.													—	—	—	—	—	—						
Final Technical Report and User's Manual																		—	—					

4. <u>Statement of Work</u>

4.1 Phase I - <u>Bi-material Cracked Element Development</u>:

The contractor will develop a bi-material cracked finite element based on a displacement hybrid finite element model. This model will require extensive modifications to the homogeneous cracked element developed under AFOSR Grant 74-2667. A modified variational principle of potential energy with relaxed requirements for continuity of displacements and tractions at the inter-element boundaries in the assumed field variables will be used. This involves a three-field variational principle, with the arbitrary interior displacements for the element, inter-element boundary displacements, and element boundary tractions as variables. The unknown in the final algebraic system of equations are the nodal displacements and the three elastic-stress-intensity factors K_I, K_{II}, and K_{III} at nodes along the crack front. Inter-element displacement compatibility will be satisfied by assuming an independent inter-element boundary displacement field and using a Lagrange Multiplier technique to enforce compatibility. These Lagrange Multipliers, which are physically the boundary tractions, are assumed from an equilibrated stress field derived from three-dimensional Beltrami or Maxwell-Morera stress functions. Considerable care must be exercised in the use of these stress functions such that the stresses produced are not linearly dependent. Since the method is based on a rigorous variational principle which enforces the conditions of inter-element displacement and traction continuity when appropriate singular stresses and displacements are included in the region of the crack front, the convergence of the finite element solution for nodal displacements as well as stress-intensity factors is established mathematically. The geometry of the basic element to be used in the bi-material development will be a 20-node isoparametric brick element with 60 degrees of freedom.

The specified tasks are addressed below.

4.1.1 Task 1. Development of asymptotically correct element-interior displacement field for bi-material element. Near-field asymptotic displacements for an embedded crack in homogeneous material have been extensively developed in literature, however, such is not the case for cracks at bi-material interfaces. However the stress field at bi-material interfaces have been studied to some extent in the literature. This task involves the analytical integration of the stress field to derive the asymptotically correct deformation field; and also to obtain these solutions directly from the differential equations where such solutions do not exist in literature.

4.1.2 Task 2. Development of asymptotically correct boundary-displacement field for bi-material element. In the hybrid-displacement finite element model, an inherently compatible element-boundary displacement field has to be assumed in addition to the interior displacement field. Thus on boundary-faces of the element which intersect or are aligned with the crack boundary, an appropriate "singular" field has to be assumed; whereas on element boundary surfaces that are away from the crack front, the field assumptions can be of regular polynomial type. The assumptions of boundary-displacement field have been found to have significant impact on the accuracy of computed intensity factors in the homogeneous element case. Thus considerable care has to be exercised in the proper choice of these functions for the bi-material case.

4.1.3 Task 3. Development of asymptotically correct boundary traction field for bi-material element: In the formulation of a bi-material crack element based on displacement hybrid model, the boundary tractions in the singular elements must also contain a singular behavior. These singular tractions as well as the non-singular traction field, which mathematically can be interpreted as Lagrange Multipliers to satisfy inter-element displacement compatibility, will be assumed from an equilibrated stress field derived from three-dimensional Beltrami-Maxwell-Morera stress functions that are complete. However, considerable care should be exercised in choosing these stress functions such that the stresses produced by any of these stress function components are not linearly dependent. Moreover the assumed stress variations must reflect the anticipated logarithmic oscillations at the bi-material interface. Research will be conducted in choosing an appropriate traction field for the bi-material cracked element.

4.1.4 Task 4. Computer coding for obtaining condensed stiffness matrix for bi-material element: Using the assumptions for the three-field variables as described in sections 4.1.1 to 4.1.3, the methodology of the hybrid-displacement model (as described in pages

1-4 of our proposal dated 10th August 1976, and elaborated in the references cited therein) will be used to obtain the condensed stiffness matrix (60x60) of a 20-node bi-material crack element. However, the computations leading to the stiffness matrix involve several novel mathematical problems. Since in the region of a crack at a bi-material inface, the stresses are singular ($1/\sqrt{r}$) and possess a logarithmic oscillation, the computation of strain-energy density corresponding to these stresses necessarily involves integrals where the integrands are singular. Thus, efficient numerical integration schemes have to be devised to evaluate these integrals to a high degree of accuracy which is necessary for the eventual accurate computation of stress-intensity factors.

4.1.5 Task 5. Verification of properties of bi-material crack-element stiffness matrix. The usual tests for the necessary properties of an element stiffness matrix such as -- (a) the satisfaction of equilibrium conditions at the element level and (b) the inherent directional symmetries and asymmetries of the elements of a stiffness matrix will be performed. If one or several of these conditions are not satisfied, necessary modifications of the assumed element functions will be made.

4.1.6 Task 6. Incorporation of thermal effects in homogeneous crack element. This task essentially involves the computation of an equivalent nodal force vector corresponding to the specified temperature distribution in the crack element. However, the computer coding has to be done carefully, so as to be compatible with the existing subroutines in TEXGAP-3D. Since the version of TEXGAP-3D currently available to us has the feature of a constant temperature level for each element, our development will be limited to this case. However, in this subroutine, the flexibility shall be kept such that at a later date, with AFRPL approval, provisions for a thermal gradient within the element can be made.

4.1.7 Task 7. Incorporation of crack face pressure for homogeneous element: This involves the proper assumptions of boundary-tractions in the hybrid-element formulation, to match those specified. The equivalent nodal force vector subroutine will be coded so as to be compatible with TEXGAP-3D.

4.1.8 Task 8. Checking compatibility of developments in Tasks 6 and 7 with TEXGAP-3D: This primarily involves running reasonable size three-dimensional crack problems involving thermal loading and crack pressure distribution to verify that the execution of the complete TEXGAP-3D subroutines (with the developments of Tasks 6 and 7 incorporated in them) is successful; and that the data and variables used in the singular elements do not cause interruption of the execution of the complete TEXGAP-3D program.

4.1.9 Task 9. Repetition of Tasks 6, 7, and 8 for bi-material crack element: This involves primarily the incorporation and checking compatibility with TEXGAP-3D of thermal and crack pressure capabilities into the bi-material cracked element.

4.2 Phase II - Incorporation of Cracked Elements in TEXGAP-3D

The geometry of the homogeneous and bi-material basic "elements" are similar to the "regular elements" already built into TEXGAP-3D and can thus be directly incorporated into the code. By incorporating the statically condensed stiffness matrix of the cracked elements in TEXGAP-3D, the solution procedure for the mixed mode stress-intensity factors is considerably simplified. Programming techniques which are conservative of computer time and storage requirements will be utilized.

The specific tasks are addressed below.

4.2.1 Task 1. Optimization of coding for existing homogeneous element: The homogeneous cracked element, developed under AFOSR Grant 74-2667, was primarily developed as a theoretical research tool, and as such its development did not involve optimization of computing time. The objective of this task is to employ programming techniques that will results in saving of computer time to generate the condensed stiffness matrix of the crack element.

4.2.2 Task 2. Incorporation of homogeneous element (without thermal and pressure effects) into TEXGAP-3D: This task primarily involves the incorporation of the "condensed stiffness matrix" subroutine for the homogeneous crack element into TEXGAP-3D and to verify that the execution of the entire TEXGAP-3D for simple 3-D fracture problems is successful. As a first step, however, thermal and crack-pressure effects are ignored.

4.2.3 Task 3. Incorporation of homogeneous element with thermal and crack-pressure effects into TEXGAP-3D: This task involves the repitition of Task 2 and execution of simple homogeneous material fracture problems involving thermal loading and crack pressure distribution, using the entire TEXGAP-3D program, with the homogeneous crack elements.

4.2.4 Task 4. Incorporation of bi-material element with thermal and pressure effects into TEXGAP-3D: This involves repitition of Task 3, with the bi-material crack element, and execution of simple bi-material fracture problems using TEXGAP-3D.

4.3 Phase III - Verification

4.3.1 Task 1. Verification problems for homogeneous element (with thermal and crack pressure effects): This involves execution of realistic, large-scale problems of homogeneous material fracture using the homogeneous crack elements, and with thermal and crack pressure effects. The successful execution of these problems will verify all the features of development including data input, output, and stress-intensity computations.

4.3.2 Task 2. Verification problems for bi-material element. This involves the execution of realistic, large-scale problems of bi-material fracture using the bi-material crack elements. The successful execution of these problems will verify all the features of development pertaining to the bi-material element, including data input, output, and stress-intensity computations.

4.4 Final Technical Report and User's Manual

A detailed user's manual and documentation will be provided for the homogeneous and bi-material elements to enable the user to set up, model, solve, and assess the results of typical crack problems which are relevant to the solid propellant industry. This user's manual documentation for the cracked elements, coupled with the detailed user's manual documentation for TEXGAP-3D, will provide the necessary tools to solve complex fracture mechanics problems in solid propellant rocket motors.

The user's manual will thus contain several classical problems with cracks in homogeneous materials as well as at bi-material interfaces for purposes of verifying the code on the different computer systems in the industry. These problems will be illustrated in detail and input and output approaches will be documented.